Money as a Unit of Account*

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Abstract

We develop a theory that rationalizes the use of a dominant unit of account in an economy. Agents enter into non-contingent contracts with a variety of business partners. Trade unfolds sequentially in credit chains and is subject to random matching. By using a dominant unit of account, agents can lower their exposure to relative price risk, avoid costly default, and create more total surplus. We discuss conditions under which it is optimal to adopt circulating government paper as the dominant unit of account, and the optimal choice of "currency areas" when there is variation in the intensity of trade within and across regions.

Keywords: unit of account, credit chains, balance sheet risk, currency areas

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1 Introduction

Classical economists pointed out money's various functions in society. Since then, large literatures have rationalized the use of money as a store of value and as a medium of exchange.[1] In contrast, the use of money as a unit of account for future payments has received little attention. This fact is surprising given the widespread use of money-denominated long-term contracts (such as bonds and mortgages) in modern economies. The use of money as a unit of account implies that inflation has redistribution effects, which lie at the heart of Irving Fisher's debt-deflation theory of depressions (Fisher 1933) and which are just as relevant today.

The goal of this paper is to explain the role of money as a unit of account for future payments. At first sight, the use of money as a unit of account might appear to be a matter of convenience only. If future payments will be settled in money anyway (since it is the medium of exchange), isn't it practical to specify the value of the payments in terms of money as well (as the unit of account)? While such an explanation may seem straightforward in modern economies where the same money serves both functions (such as in the United States, where the dollar is both the dominant medium of exchange and the dominant unit of account), monetary history offers numerous examples where the medium-of-exchange and unit-of-account functions do not coincide.

Indeed, in medieval Europe a separation of the different functions of money was the rule rather than the exception (see Spufford 1988 and Kindleberger 1993 for overviews of European monetary history). In France, for example, the *livre tournois* served as unit of account for centuries during the medieval and early modern periods, even when the corresponding coin was no longer in circulation. The value of a coin used as an accounting currency could also be different from that of the same coin in circulation, a phenomenon referred to as "ghost

[1] Two seminal contributions are Samuelson (1958) on money as a store of value in an overlapping-generations model, and Kiyotaki and Wright (1989) on money as a medium of exchange in a model with search frictions.

money" by Cipolla (1956) and "imaginary money" by Einaudi (1937, 1953).[2] In Germany, specific coins (such as the *Vereinsthaler*) were used as an accounting currency across large areas, even though different media of exchange circulated in the various sovereign states of Germany. Also common was the use of natural units in contracts (such as bushels of wheat) and of bundles (such as a combination of a natural unit and a monetary unit). A more recent example of a unit of account that is not also a medium of exchange is the *ECU* (European Currency Unit), which was based on a basket of European currencies and served as a unit of account in European trade before the introduction of the euro. A practice that is still common today is the use of foreign currency as a unit of account, such as the use of Italian *Florin* in medieval Europe, the modern use of the U.S. dollar in trade relationships not involving the United States, and mortgage borrowing denominated in euros or Swiss francs in Eastern Europe.

In light of these observations, we address two separate questions on the role of money as a unit of account. First, why do economic agents often find it useful to coordinate on a dominant unit of account? Second, what should a useful unit of account look like? The answer to the first question should also address the limits of coordination: why do different groups of people sometimes use different units of account, for example by forming currency areas? The answer to the second question should explain in particular the emergence of government-issued money as a unit of account: Why is it often the dominant unit of account in modern times, but was less so in earlier times? And why are the different functions of money not always linked (as in medieval Europe, and in modern countries where private contracts are dollarized)?

Our theory is based on four features shared by most economies. First, agents enter into contracts that involve payment promises that are later costly to break or renegotiate. Second, there are multiple widely traded goods in which promised payments can, in principle, be denominated, and which are subject to fluctuating

[2]Even where a circulating coin was used as unit of account, its use in contracts was solely as a specification of value: "...it was tacitly assumed that the payment could be settled with any other commodity of equivalent value. A debt stipulated in 20 *solidi* in a French document of November 1107 was, we know from a later document, settled with a horse" (Cipolla 1956, p. 5; see also the discussion in Sargent and Velde 2002, p. 126–128).

prices. These goods can be interpreted broadly, for example as currencies, precious metals, or government paper. In this setting, a contract between a lender and a borrower has to specify the unit of account, i.e., the good in which the value of future payments is specified. The cost of breaking promises along with price risk implies that borrowers can gain from using the same unit of account on both sides of the balance sheet. Specifically, if the price realization of a good that denominates a large part of a borrower's income is low, the borrower may have difficulty meeting his own promises. This risk can be hedged by denominating outgoing payments in the same good that denominates borrower income.[3]

To give an example of the balance-sheet risk that we have in mind, consider an economic agent (such as a household, a firm, or a bank) who holds assets that are denominated in U.S. dollars. In other words, the agent expects to receive future payments, the value of which is fixed in terms of dollars. Now suppose that the agent wants also to incur liabilities, such as by borrowing in order to invest in a business or buy a house. If these liabilities are denominated in a unit of account other than the U.S. dollar (say, euros), the agent faces the risk that the relative price of the units of account for assets and liabilities will change until future payments are due. Here, the risk is that the price of euros will rise relative to dollars. If there is a big change in the relative price, the value of the assets (the future payments in terms of dollars) may be too low to repay the liabilities (in terms of euros), resulting in costly default. By using the same unit of account for both assets and liabilities, the agent can avoid this relative-price risk and thereby lower the probability of default.

Hence, the first two elements of our theory, the cost of breaking promises and price risk, explain the demand for specific units of account. The third element of our theory is that efficient production requires an entire network of borrowing and lending relationships. As a result, a typical agent is both a borrower and a lender—he is a member of a credit chain. Credit chains arise naturally in modern

[3] We focus on the unit-of-account function of money for future payments precisely because the delay between making the promise and the actual payment implies the possibility of relative-price changes. In contrast, the unit-of-account function of money for quoting current prices is not subject to the same price risk. However, the unit of account for current prices may still matter if additional frictions are present, such as a cost of changing prices (from which we abstract here).

economies not only in the organization of production (for example, raw materials, intermediates, and final-goods producers), but also in commerce (producer, wholesaler, retailer) and finance (borrower, intermediary, investor). The presence of credit chains explains the propagation of units of account beyond bilateral relationships. In a credit chain, what is a promise for one agent is income for the next, thus leading to demand for a common unit of account in the entire chain.

The final element of our theory is that the formation of credit chains has a random-matching component that is not contractible. When a borrower and a lender meet, they typically cannot condition payment on the identity of their future business partners, let alone those partners' partners and so on. Balance sheet risk therefore comes from two sources. In addition to variation in relative prices there is matching uncertainty, as agents do not know which credit chain they will ultimately be part of. It then becomes advantageous to adopt a unit of account that is likely to be compatible with many future potential trading partners, leading to the optimality of an economywide dominant unit of account.

The nature of the efficiency gain from adopting a dominant unit of account depends on how costly it is to break promises. In the main part of the analysis, we consider the extreme where breaking promises is infinitely costly. In this setting, contracts are written so that all parties are always able to meet their promises, and default never occurs. To ensure that default can be avoided, borrowers lower debt ex ante, which leads to inefficiently low production. Use of a dominant unit allows more borrowing and thereby more production. In the online supplement, we also explain how our results generalize to the case where breaking promises carries only a small cost. If default costs are sufficiently small, borrowers will produce at the efficient scale and default if necessary. Use of a dominant unit of account then lowers average ex-post default costs.

The argument we have outlined so far explains why agents coordinate on a common unit of account, but leaves open the question of exactly what should be the unit of account. Given that in our theory a key role of the unit of account is to minimize balance-sheet risk, choosing a unit of account that already denominates the income of major borrowers is often useful. This observation suggests a tight link between the use of government-issued money as unit of account and the issuance of government debt. Consider a government that issues money-denominated (i.e., nominal) bonds to be held by households, firms, and banks. The payments promised in the bonds are part of these agents' future income. If the same agents now incur future liabilities, they can reduce their balance-sheet risk by denominating these liabilities in money also. Thus, the government's use of money for its own borrowing propagates to private contracts and leads to money being the dominant unit of account.

Notice that our argument explaining the use of circulating money as the unit of account relies solely on the role of money as denominating government debt, but not on the medium-of-exchange role of money. Indeed, our theoretical model features a centralized spot market in which there is no need for a specialized medium of exchange. The implication of a link between government debt and the unit of account is consistent with the observation that, historically, units of account and media of exchange often used to be distinct, but became unified in modern economies characterized by the widespread use of government-issued nominal bonds.

In our theory, there are additional factors (other than the presence of government debt) that determine the optimal unit of account. For example, it is useful for a unit of account to be stable in value relative to other goods traded in the economy. This feature explains why if the value of money is too volatile (i.e., volatile inflation), local currency may fail to be used as a unit of account even if nominal government debt is present. Such a scenario is akin to the dollarization of private contracts that is often observed in countries grappling with high inflation. In addition, different regions or countries may have different dominant income sources. This scenario leads to a tradeoff between a unified unit of account versus multiple units that may be better suited to local conditions i.e., a theory of optimal currency areas.

The paper is structured as follows. In the following section, we relate our work to the existing literature. In Section 3 we consider the optimal use of units of account in bilateral contracts. In Section 4, we introduce credit chains and random matching to account for the use of a dominant unit of account throughout an entire economy. In Section 5, we discuss conditions under which government-issued paper (such as money) may arise as the optimal unit of account. In Section 6, we apply the model to the issue of optimal currency areas. Section 7 concludes. Proofs for propositions are provided in Appendix A, and extensions of the basic model setup are discussed in an online supplement.

2 Related Literature

Our paper is related to existing work on balance sheet effects of price changes. The basic idea that mismatched units of account on a balance sheet can create problems is familiar from the banking literature, and currency mismatch has played an important role in banking and financial crises (see for example Schneider and Tornell 2004 and Burnside, Eichenbaum, and Rebelo 2006). In this paper, we go beyond individual balance sheets and find conditions under which a dominant unit of account will be adopted in an entire economy. Relative to the banking literature, the key features that lead to this result are that production takes place in chains of credit (modeled as in Kiyotaki and Moore 1997) and that contracting is non-synchronized.

Our work is also related to a small literature on the optimality of nominal contracts. Jovanovic and Ueda (1997) consider a static moral hazard problem in which nominal output is observed before the price level (and therefore real output) is revealed. In addition, contracts are not renegotiation-proof, so that principal and agent have an incentive to renegotiate after nominal output is observed. In the optimal renegotiation-proof solution, the principal offers full insurance to the agent once nominal output is known. This implies that the real wage depends on nominal output, so that the contract can be interpreted as a nominal contract. Meh, Quadrini, and Terajima (2010) integrate a similar mechanism into a model with firm heterogeneity and financial constraints, and study how different monetary regimes affect the degree of indexation. The mechanism in these papers operates within a relatively short time horizon, namely the lag between the realization of a nominal variable and the observation of the corresponding price level. In contrast, the balance-sheet effects in our theory are equally relevant for long-term assets such as bonds and mortgages, which account for the major part of redistribution effects of inflation.

Freeman and Tabellini (1998) consider an overlapping-generations economy with spatially separated agents in which fiat money serves as a medium of exchange. They provide conditions for fiat money to serve also as a unit of account. In contrast, in our theory exchange takes place in a frictionless centralized market, without a need for a medium of exchange. Rather, in our theory the occasional coincidence between the unit of account and the medium of exchange happens only in the presence of government debt denominated in fiat currency. Our approach has the advantage that it can explain why, in modern economies with widespread use of government debt, it is common for fiat money to serve both functions, whereas in earlier times distinct monetary units were used as unit of account and medium of exchange.

Cooper (1990) and Acemoglu (1995) also consider environments in which each agent writes several contracts within a fixed network. They assume uncertainty about the value of money and provide conditions for the coexistence of multiple equilibria with indexed or nominal contracts. For example, both equilibria exist if the network of contracts is such that coordination on a single unit of account provides perfect hedging. Our setup has multiple goods (and hence multiple sources of price risk), so that a fixed network does not give rise to a dominant unit of account. Instead, random matching is critical both for making a dominant unit of account optimal and for determining what that unit looks like.

Our model shares several features with the literature on microfoundations for media of exchange that follows the seminal work of Kiyotaki and Wright (1989). In particular, a key common element is that there are gains from coordinating payment across many pairwise meetings. In the typical model of decentralized exchange, there is no double coincidence of wants in individual meetings. Gains from trade are realized by passing the medium of exchange along from one meeting to the next. In our model, there are gains from making credit chains longer by adding additional producers. Those gains are realized by choosing the unit of account so funds can be passed along the credit chain without default once income risk is realized and credit contracts are settled.

Another prominent element in models of media of exchange is random matching. Compared to our environment, however, the role of random matching in those models is quite different. Models of media of exchange typically assume that agents cannot enforce contracts written in past meetings. Random matching then ensures that agents do not meet each other again. As a result, credit becomes infeasible and money becomes essential (for example, Kocherlakota 1998). In our setup, contracts can be enforced. The role of random matching is to make borrowers' income risk similar across many bilateral contracting relationships, which implies the emergence of a dominant unit of account.

Within the literature on media of exchange, Lagos and Wright (2005) introduced a structure in which both decentralized and centralized markets play a role, as in our setup. However, in Lagos and Wright (2005) exchange takes place in both markets, and the centralized market mainly serves to offset the heterogeneity that is generated by random matching. In contrast, in our theory random matching affects only the contracting stage, and all contracts are ultimately settled in the centralized market.

In most of the paper, we employ a normative approach that derives properties of second best allocations, rather than spell out a particular game that describes trade in the economy. A number of papers on media of exchange also follow this strategy, for example Kocherlakota (1998, 2002), Cavalcanti and Wallace (1999), Hu and Rocheteau (2013), and Hu, Kennan, and Wallace (2009). A difference in approach is that those papers tend to make no a priori restrictions on contracts. In contrast, a key friction in our setup is that contracts are noncontingent. In this regard, our model is closer to the literature on general equilibrium with exogenously incomplete markets that also studies welfare properties of a given asset structure. An interesting question for future research is to study the unit of account in an economy in which incompleteness arises endogenously from limited commitment or asymmetric information. Such an approach may also shed further light on why the same asset often serves as the medium of exchange as well as the unit of account.

The literature on media of exchange also studies the government as a large player (Aiyagari and Wallace 1997, Li and Wright 1998). Our results on government debt as a unit of account are related to results on how the medium of exchange is affected by what the government accepts in transactions. In both cases, the size of government is a key parameter. What is different is how one would measure size in each case. In models of media of exchange, what matters is the share of transactions with the government, either due to government purchases or the collection of taxes (Starr 1974 and Goldberg 2012 provide models of the "tax foundation" theory of money as a medium of exchange). In our context, the appropriate measure of the size of government is the effect of government liabilities on borrowers' ability to pay.

Another important theme in our setup is that the optimal unit of account should not be subject to large price spikes. Models of media of exchange also point to stability in value as a property that may select the optimal medium among several objects. For example, Banerjee and Maskin (1996) and Rocheteau (2011) study environments in which goods or assets are subject to asymmetric information about quality. The object with the smallest conditional volatility in quality then emerges as the medium of exchange. In contrast to these papers, information in our setup is symmetric. The possibility of price spikes make a unit of account unattractive only if they cannot be hedged due to the incompleteness of contracts.

Finally, our work relates to the literature on the redistribution effects of inflation. Most of this literature focuses on a particular aspect of redistribution, namely the revaluation of government debt (see for example Bohn 1988, 1990, Persson, Persson, and Svensson 1998, Sims 2001). Government debt plays an important role in our model also, in a mechanism that renders fiat money an attractive choice for the unit of account. Redistribution effects among private agents were recently considered by Adam and Zhu (2016), Auclert (2016), Coibion et al. (2012), Doepke and Schneider (2006a), Doepke and Schneider (2006b), Doepke, Schneider, and Selezneva (2016), Meh, Ríos-Rull, and Terajima (2010), and Sterk and Tenreyro (2014).

3 Income Risk and the Optimal Unit of Account in Bilateral Contracts

In this section, we analyze a bilateral contracting problem. The analysis shows why the unit of account matters in contracting, and it isolates forces that determine the optimal unit of account. In the following section, we expand the analysis to a general-equilibrium model with many contracting relationships, in order to understand why we often observe coordination on a dominant unit of account in an entire economy.

3.1 Environment: Dates, Goods, and Preferences

We consider a bilateral contracting problem between the supplier of a customized good and his customer. There are three dates, 0, 1, and 2. At date 0, the supplier and customer can write a contract, to be specified below. At date 1, the supplier can expend $x > 0$ units of labor effort to make x units of the customized good, available for consumption by the customer at date 2. Only the customer benefits from the customized good—it cannot be consumed by the supplier and it cannot be sold in a market.

Also at date 2, a spot market opens in which two goods A and B can be traded, which may provide utility to the supplier and the customer. The price vector $\boldsymbol{p}$ in the spot market is exogenous and random, with convex and compact support $\boldsymbol{P} \subset \mathbb{R}^2_{>0}$, where $\mathbb{R}_{>0}$ denotes the set of strictly positive reals. When the spot market opens, the customer receives a random endowment vector $\boldsymbol{y}$ of these traded goods, drawn from a distribution with compact support $\boldsymbol{Y} \subset \mathbb{R}^2_{\geq 0}$, where $\mathbb{R}_{\geq 0}$ denotes the set of nonnegative reals. Both supplier and customer can access the spot market.

Our assumption that the support of the endowment vector is unrelated to that of prices simplifies the analysis. In principle, one could imagine that the support of prices changes conditional on the realization of the endowment. Our assumption is natural if endowment risk is primarily idiosyncratic. We emphasize that the assumption only restricts supports: for given support, we allow prices and endowments to be correlated in arbitrary ways. Convexity of the support of prices is helpful to establish uniqueness of the optimal unit of account below.

The utility function of a supplier who works x units of time and consumes a vector $\boldsymbol{c}$ of tradable goods is:

$$v(\boldsymbol{c}) - x,$$

where $v(\cdot)$ is homogeneous of degree one. The utility of a customer who receives x units of the customized good and consumes a vector $\boldsymbol{c}$ of tradable goods is:

$$v(\boldsymbol{c}) + (1 + \lambda)\, x.$$

We assume that $\lambda > 0$ so that there are gains from trade, i.e., both parties can be made better off by a transfer of tradable goods from the customer to the supplier in exchange for the customized good x produced using the supplier's effort.

It is convenient to normalize prices of tradable goods and the units in which goods are measured such that the utility derived from one unit of income in the spot market is one, and such that the expected price of each good $i \in \{A, B\}$ is one.

Assumption 1 (Normalization of Prices)**.** *Let $\tilde{P}(\boldsymbol{p})$ be the expenditure function at a utility level of one, that is:*

$$\tilde{P}(\boldsymbol{p}) = \min_{\boldsymbol{c}} \{\boldsymbol{p}'\boldsymbol{c}\}$$

subject to $v(\boldsymbol{c}) \geq 1$. Prices are normalized such that the price vector $\boldsymbol{p}$ satisfies:

$$\tilde{P}(\boldsymbol{p}) = 1 \quad \textit{for all} \quad \boldsymbol{p} \in P.$$

In addition, units are chosen such that the expected price of each good $i \in \{A, B\}$ equals one:

$$E(p^i) = 1.$$

Setting $\tilde{P}(\boldsymbol{p}) = 1$ is without loss of generality, because only relative prices matter. Starting from any initial price distribution $\tilde{\boldsymbol{p}}$ with $\tilde{P}(\tilde{\boldsymbol{p}}) \neq 1$, we can rescale prices as $\boldsymbol{p} = \tilde{\boldsymbol{p}}/\tilde{P}(\tilde{\boldsymbol{p}})$ for each $\tilde{\boldsymbol{p}}$ to meet the condition. The assumption that $E(p^i) = 1$

is not a simple normalization of the price vector. It is nevertheless innocuous: it amounts to a choice of the units in which each tradable good is measured.[4]

Given that v is homogeneous of degree one, the normalization $\tilde{P}(\boldsymbol{p}) = 1$ implies that the indirect utility (in terms of consuming market goods) of an agent who owns tradable goods $\boldsymbol{y}$ at the beginning of date 2 is simply $\boldsymbol{p}'\boldsymbol{y}$.

3.2 Contracts

The timing of the environment implies that when supplier and customer meet at date 0 to agree on a contract, a need for credit arises. The supplier must work at date 1 if he is to deliver the customized good at date 2. However, at date 0 the customer does not have any tradable goods that could be used to pay for the customized good up front. Rather, any payments have to take place at date 2, after uncertainty regarding the customer's endowment $\boldsymbol{y}$ and the price realization $\boldsymbol{p}$ has been resolved. A contract specifies a quantity of the customized good to be delivered from supplier to customer together with a payment in terms of tradable goods from the customer to the supplier.

The key contracting friction that underlies our analysis is that contracts involve simple, non-contingent payment promises. Specifically, a payment promise consists of a bundle $\boldsymbol{\pi} \in \mathbb{R}^2_{\geq 0}$ of the market goods. The bundle is agreed on at date 0 before uncertainty is resolved. The payment promise cannot be made contingent on the realization of the customer's endowment $\boldsymbol{y}$ or the vector of tradable goods prices $\boldsymbol{p}$. In our main analysis, we assume that an agent who defaults on a promise faces an arbitrarily large punishment, so that all payment promises that are made are kept. In Appendix D in the online supplement, we show how our results can be generalized to a setting where the cost of breaking promises is positive but finite, so that there is an incentive to choose contracts with a high likelihood that promises will be kept. The contracting friction can be motivated by legal costs of interpreting and enforcing complicated contracts, and fits well with the observation that most real-world contracts indeed involve simple promises.

[4] For example, if for good A we have $E(p_A) = 2$, we can divide the unit of measuring good i by two (i.e., multiply endowments by two and modify the utility function by dividing consumption of good i by two where it enters utility) to yield an expected price of one.

The full contract between supplier and customer specifies the artisanal goods x to be produced by the supplier at date 1 and to be delivered to the customer at date 2, and the payment promise $\boldsymbol{\pi}$. By promising $\boldsymbol{\pi}$ to the supplier, the customer commits to delivering goods $\boldsymbol{\pi}$ at date 2. Since the promised goods can be freely exchanged in the spot market, the commitment is effectively to the value $\boldsymbol{p}'\boldsymbol{\pi}$. Since only the value of the payment matters, there is no need to settle the contract in the goods in which it is specified.

In the second period, the customer must be able to make the promised payment for all possible realizations of endowments and prices. A feasible payment thus satisfies

$$\boldsymbol{p}'\boldsymbol{\pi} \leq \boldsymbol{p}'\boldsymbol{y} \quad \text{for all} \quad \boldsymbol{p} \in \boldsymbol{P} \text{ and } \boldsymbol{y} \in \boldsymbol{Y}. \tag{1}$$

Our focus on noncontingent contracts implies that the distribution of endowments $\boldsymbol{y}$ and prices $\boldsymbol{p}$ matters for feasibility only via their supports $\boldsymbol{P}$ and $\boldsymbol{Y}$. Compactness of $\boldsymbol{P}$ and $\boldsymbol{Y}$ implies that the set of feasible payments $\boldsymbol{\pi}$ is compact. A feasible contract $(x, \boldsymbol{\pi})$ specifies production of artisanal goods x together with a feasible payment.

3.3 The Unit of Account

The assumptions on contracting now allow us to discuss the use of units of account. In the customer-supplier relationship, the unit of account used in contracting is given by the bundle of market goods that denominates the payment promise $\boldsymbol{\pi}$. For example, whenever in a given contract we have $\pi^A > 0$ and $\pi^B = 0$, we say that good A serves as the unit of account. In this case, the value of the promise is specified in terms of units of good A, just as in the U.S. economy most future payments are in terms of U.S. dollars. It is also possible that a non-degenerate bundle serves as the unit of account. In the real world, this case would correspond to a contract that specifies payments in two different currencies or commodities, such as U.S. dollars and euros.

To formalize the notion of a unit of account, we can decompose any payment $\boldsymbol{\pi}$ into a scalar *scale* of the payment $q(\boldsymbol{\pi})$ and the *unit of account* of the payment $\boldsymbol{u}(\boldsymbol{\pi})$, where $0 \leq u^i \leq 1$ and $\sum_i u^i = 1$. Here $i \in \{A, B\}$ denotes the goods included in the payment. For a given payment $\boldsymbol{\pi}$, $q(\boldsymbol{\pi})$ and $\boldsymbol{u}(\boldsymbol{\pi})$ can be computed as:

$$q(\boldsymbol{\pi}) = \sum_i \pi^i, \tag{2}$$

$$\boldsymbol{u}(\boldsymbol{\pi}) = q(\boldsymbol{\pi})^{-1}\boldsymbol{\pi}, \tag{3}$$

so that the payment is given by $\boldsymbol{\pi} = q(\boldsymbol{\pi})\boldsymbol{u}(\boldsymbol{\pi})$.

We refer to the vector $\boldsymbol{u}(\boldsymbol{\pi})$ as the unit of account because it determines in which goods the payment is denominated. The payment in terms of good $i \in \{A, B\}$ is given by $q(\boldsymbol{\pi})u^i$. If we have $u^A = 1$ and hence $u^B = 0$, the payment is entirely in terms of good A, so that good A serves as the unit of account. If $u^A = u^B = 0.5$, the unit of account is an equally weighted bundle of goods A and B.

Recall from Section 3.1 that the indirect utility from consuming market goods of a consumer who owns tradable goods $\boldsymbol{\pi}$ is given by $\boldsymbol{p}'\boldsymbol{\pi}$. Given Assumption 1 (i.e., $E(p^i) = 1$), the expected indirect utility derived from owning $\boldsymbol{\pi}$ is $\sum_i \pi^i = q$. That is, utility from consuming market goods depends only on the scale of a payment received, but not on its composition (i.e., the unit of account). The parties' expected utility from a contract $(x, \boldsymbol{\pi})$ can now be written as:

$$U_S\left(x, \boldsymbol{\pi}\right) = q(\boldsymbol{\pi}) - x, \tag{4}$$

$$U_C\left(x, \boldsymbol{\pi}\right) = E(\boldsymbol{p}'\boldsymbol{y}) - q(\boldsymbol{\pi}) + (1 + \lambda)\, x, \tag{5}$$

where U_S is the utility of the supplier and U_C the utility of the customer.

Our assumptions on technology and preferences imply that a higher scale of payment together with proportionally more production of customized goods always results in a Pareto improvement. Indeed, if we increase x one-for-one with q, supplier utility is unchanged, whereas customer utility increases by λx. Hence, optimal contracts allow as much production of the customized good x as possible. The only limit to production is the scale of payment that the customer can afford, as described by the payment feasibility constraint (1).

3.4 Why the Unit of Account Matters

We would like to examine the implications of our contracting model for the optimal use of units of account. To see how choosing the wrong unit of account may reduce the maximal feasible scale of payment and hence welfare, consider a simple example.

Example 1. *The customer has a certain endowment $\boldsymbol{y}$ of one unit of good A (and no endowments of other goods).*

If good A is chosen as unit of account—that is, $u^A = 1$—then the maximal scale of payment that satisfies constraint (1) is $q = 1$. In this case, the customer can promise the supplier his entire endowment. Now suppose instead that the unit of account is good B instead. With $u^A = 0$ and $u^B = 1$, constraint (1) can be written as:

$$p^B q \leq p^A \quad \text{for all} \quad p^B, p^A.$$

On the left-hand side is the value of the promised payment in the centralized market, and on the right-hand side is the value of the endowment.

Since the constraint has to hold for all prices, the maximal feasible scale of payment $q(\boldsymbol{\pi})$ is:

$$q = \min_{\boldsymbol{p}} \left\{ \frac{p^A}{p^B} \right\}.$$

Given that $E(p^i) = 1$ for all $i \in \{A, B\}$, the right-hand side cannot be bigger than one. To the contrary, if the price of good A relative to good B is ever below one, then the maximal feasible scale of payment is below one also. The maximal payment will be especially low if good B is subject to large price spikes (high p^B).

The example shows that the underlying problem is relative price risk: the payment is constrained by the possibility that the price of the goods that constitute the promised payment is high, whereas the price of the goods that make up the endowment is low. By choosing the unit of account judiciously, this relative price risk can be hedged. In the example considered here with a certain endowment in terms of a single good, the price risk can be hedged perfectly by promising the same good that makes up the endowment.

More generally, even if perfect hedging is not possible, the choice of unit of account still determines how much can be promised. Consider the following example:

Example 2. *The customer receives a certain endowment $\boldsymbol{y}$ of one unit of some good (A or B), but the identity of the good is random.*

The customer must now be able to make payment regardless of the identity of the good he receives. To allow a scale q of one, the value of the unit of account $\boldsymbol{p}'\boldsymbol{u}$ always would have to be at least as high as every individual price p^i. Since $E(p^i) = 1$ and prices fluctuate, this is impossible. It follows that the maximal scale of payment is below one, and hence does not always exhaust the endowment. The reduction in the scale of the payment is due to the contracting problem.

3.5 The Optimal Unit of Account

We now state the contracting problem formally and show that it reduces to finding payments that maximize the scale of payment $q(\boldsymbol{\pi})$ subject to feasibility (1). Our approach is to characterize the Pareto frontier of contracts, subject to the payment feasibility constraint and to individual rationality constraints stating that each party is no worse off from entering the contract. By characterizing the entire Pareto frontier, we do not have to take a stand on the details of bargaining between the parties and on how potential surplus is distributed.

A contract $(x, \boldsymbol{\pi})$ is individually rational if the contract makes both parties better off than autarky. Left on his own, the supplier would get zero utility. The customer would derive indirect utility from the endowment $\boldsymbol{y}$, so that the individual rationality constraints are:

$$U_S\left(x, \boldsymbol{\pi}\right) \geq 0, \tag{6}$$

$$U_C\left(x, \boldsymbol{\pi}\right) \geq E(\boldsymbol{p}'\boldsymbol{y}). \tag{7}$$

We are now ready to provide a formal definition of the optimality of contracts.

Definition 1 (Optimality of Bilateral Contracts)**.** *A contract* $(x, \boldsymbol{\pi})$ *is (constrained Pareto) optimal if for some* $v_S \geq 0$ *it maximizes* $U_C(x, \boldsymbol{\pi})$ *subject to* (1) *(i.e., the payment* $\boldsymbol{\pi}$ *is feasible),* (6) *and* (7) *(i.e., the contract is individually rational), and* $U_S(x, \boldsymbol{\pi}) = v_S$*.*

Lemma 1 (Optimal Contract Maximizes Scale)**.** *Any optimal contract* $(x, \boldsymbol{\pi})$ *includes a payment that maximizes the scale of payment* $q(\boldsymbol{\pi})$ *subject to feasibility* (1)*.*

Proof: From (4), in order to promise the supplier utility $U_S(x, \boldsymbol{\pi}) = v_S$, production of customized goods must be given by $x = q(\boldsymbol{\pi}) - v_S$. Substituting into the customer's utility (5), we obtain:

$$U_C(x, \boldsymbol{\pi}) = E(\boldsymbol{p}'\boldsymbol{y}) + \lambda q(\boldsymbol{\pi}) - (1 + \lambda) v_S, \tag{8}$$

which is strictly increasing in $q(\boldsymbol{\pi})$. □

Once we know the payment that maximizes $q(\boldsymbol{\pi})$ subject to feasibility (1), we can trace out the entire Pareto frontier. From the individual rationality constraint (7), the customer's utility at an optimal contract (8) must be at least $E(\boldsymbol{p}'\boldsymbol{y})$. It follows that an optimal contract can promise the supplier any utility value between zero and $\bar{v}_S \equiv \lambda q(\boldsymbol{\pi}) / (1 + \lambda)$. Along the Pareto frontier, the optimal payment $\boldsymbol{\pi}$ and hence the unit of account are always the same—all that changes is the production of customized goods. Building on Lemma 1, we now turn to characterizing the optimal payment as a function of price risk and endowment risk.

Simple Income Risk

Choosing the payment is easy if there is no endowment risk, that is, if the set $\boldsymbol{Y}$ is a singleton. The optimal payment is equal to the customer's certain endowment. Example 1 above considers the case a certain endowment of a single good. More generally, if the customer has a certain endowment of both good A and good B, the optimal payment is the bundle of these two endowments.

The presence of endowment risk may reduce scale for two reasons. First, the scale of the endowment may be uncertain. Second, the interaction of endowment and price risk may prevent the customer from promising any payment that ever exhausts the endowment. As Example 2 above illustrates, this problem can arise even if the scale of the endowment is certain. We now formally separate the two scenarios and derive their implications for the optimal payment.

We say that income risk is *simple* if there is an endowment realization $\boldsymbol{y}_0 \in \boldsymbol{Y}$ such that for all $\boldsymbol{p} \in \boldsymbol{P}$ and $\boldsymbol{y} \in \boldsymbol{Y}$ we have $\boldsymbol{p}'\boldsymbol{y}_0 \leq \boldsymbol{p}'\boldsymbol{y}$. From the feasibility constraint, it is then always possible and optimal to promise the worst income realization $\boldsymbol{y}_0$ as payment:

Proposition 1 (Optimal Unit of Account with Simple Income Risk)**.**

1. *The optimal payment is an element of $\boldsymbol{Y}$ if and only if income risk is simple. In this case, the optimal payment is given by the worst income endowment realization $\boldsymbol{y}_0$. Consequently, the optimal scale and unit of account of the payment are:*

$$q(\boldsymbol{\pi}) = \sum_i y_0^i,$$
$$\boldsymbol{u}(\boldsymbol{\pi}) = q(\boldsymbol{\pi})^{-1}\boldsymbol{y}_0.$$

2. *If, in addition, there is price risk for both goods (i.e., there are price realizations $p^i < E(p^i) = 1$ and $p^i > 1$ for all i), the optimal payment is unique.*

Proof: Part 1: Suppose income risk is not simple and consider a candidate payment $\boldsymbol{\pi} \in \boldsymbol{Y}$. We then can find a joint price and endowment realization that makes the candidate payment more expensive than the endowment, that is, $\boldsymbol{p}'\boldsymbol{\pi} > \boldsymbol{p}'\boldsymbol{y}$. It follows that the candidate payment does not satisfy the feasibility constraint (1) and cannot be optimal.

Now suppose income risk is simple. Setting $\boldsymbol{\pi} = \boldsymbol{y}_0$ meets the feasibility constraint (1). To show that $\boldsymbol{y}_0$ is the optimal payment, consider some alternative payment $\tilde{\boldsymbol{\pi}} \neq \boldsymbol{y}_0$ that yields strictly higher payoff, that is, $E(\boldsymbol{p}'\tilde{\boldsymbol{\pi}}) > E(\boldsymbol{p}'\boldsymbol{y}_0)$. Such a payment is not feasible since it does not satisfy (1) for $\boldsymbol{y} = \boldsymbol{y}_0$ and the price $E(\boldsymbol{p}) \in \boldsymbol{P}$. It follows that $\boldsymbol{\pi} = \boldsymbol{y}_0$ is optimal. □

The proof of the uniqueness result in Part 2 of Proposition 1 is contained in Appendix A.

Consider some examples of simple income risk. Generalizing the case of a certain endowment, income risk is simple if the set $\boldsymbol{Y}$ is a rectangle, that is, $\boldsymbol{Y} = \left\{(y^A, y^B) \mid y^A \in [\underline{y}^A, \overline{y}^A], y^B \in [\underline{y}^B, \overline{y}^B], \right\}$. The key property is that it is possible that both lower bounds $\underline{y}^i$ are realized together. The restriction is on the support only; there are no assumptions on the correlation of the y^i. With a rectangular endowment set, the worst income realization is $\boldsymbol{y}_0 = \{\underline{y}^A, \underline{y}^B\}$. Since only the minimum endowment of each good can be promised, the result implies that the unit of account will place more weight on goods with less endowment risk.

Income risk can be simple even if the worst-case endowments of all goods cannot be realized at the same time. For an alternative (non-rectangular) example, consider

$$\boldsymbol{Y} = \left\{(y^A, y^B) \mid y^A/\gamma + y^B = 1, y^A, y^B \geq 0\right\}.$$

The endowment set is a line, with the maximum endowment of good A given by γ and the maximum endowment of good B given by one. Let $\overline{p}^B$ denote the largest possible relative price of good B:

$$\overline{p}^B = \max_{\boldsymbol{p} \in \boldsymbol{P}} \left\{ \frac{p^B}{p^A} \right\}.$$

If $\gamma > \overline{p}^B$, then income risk is simple. Indeed, even if the relative price of good B is maximal, the value of one unit of good B is still lower than that of γ units of good A. The endowment realization $\boldsymbol{y}_0 = (0, 1)$ thus delivers the lowest income at any price. In contrast to the rectangular case, the extent of price risk matters here for whether income risk is simple. In particular, we need that relative price spikes in good B are small compared to the relative quantity movements captured by γ.

General Income Risk

Consider now the general case when there is not necessarily a single worst income realization that serves as the promised payment. The optimal payment is governed by the endowment set $\boldsymbol{Y}$ and by the extremes of the distribution of the relative price of A and B. Let $\bar{\boldsymbol{p}}^i$ denote the price vector that achieves the largest relative price of good i,

$$\bar{\boldsymbol{p}}^i = \underset{\boldsymbol{p}\in\boldsymbol{P}}{\operatorname{argmax}} \left\{ \frac{p^i}{p^j} \right\} \quad \text{for} \quad j \neq i,$$

and let

$$m^i = \min_{\boldsymbol{y}\in\boldsymbol{Y}} \overline{\boldsymbol{p}}^{i\prime} \boldsymbol{y}$$

denote the lowest income when the relative price of good i is highest. Since $\boldsymbol{P}$ and $\boldsymbol{Y}$ are compact, $\overline{\boldsymbol{p}}^i$ and m^i exist for $i = A, B$. We can now characterize the optimal unit of account in terms of $\overline{\boldsymbol{p}}^A$, $\overline{\boldsymbol{p}}^B$, m^A and m^B.

Proposition 2 (Optimal Unit of Account with General Income Risk)**.** *A payment $\boldsymbol{\pi}$ that satisfies the conditions*

$$\overline{\boldsymbol{p}}^{A\prime} \boldsymbol{\pi} = m^A, \tag{9}$$

$$\overline{\boldsymbol{p}}^{B\prime} \boldsymbol{\pi} = m^B \tag{10}$$

has the largest value $\boldsymbol{p}'\boldsymbol{\pi}$ among all payments that satisfy the feasibility constraint (1) *for any $\boldsymbol{p} \in \boldsymbol{P}$. In particular, it maximizes $q(\boldsymbol{\pi})$ and hence is an optimal payment. If, in addition, there is price risk for both goods (i.e., there are price realizations $p^i < E(p^i) = 1$ and $p^i > 1$ for all i), the optimal payment is unique.*

A typical situation is displayed in Figure 1. Here the set $\boldsymbol{Y}$ consists of the two red disks: the idea is that the customer has one of two technologies, both of which are uncertain and have bounded supports. The upper and lower tangency points represents those elements of $\boldsymbol{Y}$ that achieve minimal income at the lowest and highest relative price of good B, respectively. The slopes of the tangent lines reflect those extreme relative prices. The feasible region is shaded in yellow.

The optimal payment sits at the intersection of the two lines. The optimal payment maximizes the expected value of the payment. Since the expected prices are all one, the isovalue lines slope downwards with a slope of one, while the lines bounding the feasible set have slopes given by the two extreme prices. In fact, any price vector that is not one of the two extreme prices gives rise to isovalue lines with slopes in between the slopes of the two tangent lines. The optimal

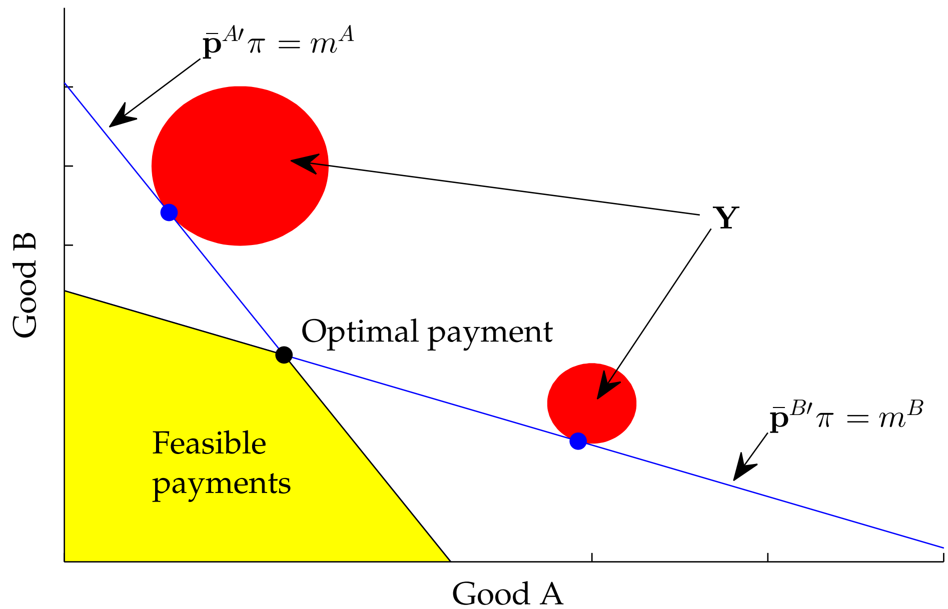

Figure 1: Determination of Optimal Unit of Account

payment thus maximizes not only value $q(\boldsymbol{\pi})$ at the expected prices, but also maximizes value at any other feasible price. This is a stronger result than what is needed to characterize the optimal payment here. It will be useful when we study general equilibrium below.

In the case of Example 1 (when the customer has a certain endowment of only good A), the endowment set $\boldsymbol{Y}$ would be a single point on the horizontal axis. This point is also where the two tangency lines intersect, and hence the optimal payment is equal to the endowment. Figure 2 displays the case where $\boldsymbol{Y}$ is a rectangle, which is a case of simple income risk as analyzed in Proposition 1. Given that the slope of the tangent lines is negative, the intersection and hence the optimal payment is at the lower-left corner of the rectangle, that is, the worst income realization for both goods.

Proposition 2 has sharp implications for how changes in income or price risk affect the optimal unit of account. Let $\boldsymbol{\pi} = (\pi^A, \pi^B)$ be the optimal unit of account given $\boldsymbol{P}$ and $\boldsymbol{Y}$. Consider, first, a shift in endowments such that the endowment of good A moves up by $\epsilon > 0$ in all states, i.e., the new endowment set is $\boldsymbol{Y}' =$

$\{(y^A, y^B) \mid (y^A - \epsilon, y^B) \in \boldsymbol{Y}\}$. It then follows that the new optimal payment is given by $(\pi^A + \epsilon, \pi^B)$, that is, the extra endowment is added to the payment, and the weight u^A on good A increases accordingly. This result shows that the optimal unit of account places more weight on goods with higher endowments. Next, consider a change in the price distribution that increases the maximum relative price of good A, $\max\{p^A/p^B\}$, with no change in the maximum relative price of good B. The new optimal payment still has to lie on the line given by (10), whereas the line given by (9) becomes steeper. If there is a change in the optimal payment, it therefore has to lie to the left of the original optimal payment on the line defined by (10), implying that u^A decreases and u^B increases. This result demonstrates that the optimal unit of account places less weight on goods subject to larger price spikes.

It is interesting to ask what changes with more than two goods. Proposition 1 readily generalizes to any finite number of goods. With simple income risk, it is always best to promise the endowment realization that delivers the lowest income for sure. Even if income risk is not simple, it remains optimal to maximize scale subject to feasibility, as in Lemma 1. However, the shape of the feasible set in multiple dimensions is more complex. As long as the optimal payment lies on the boundary of the feasible set, the support of the price set typically matters for the unit of account.

Suppose for example that the convex support of the price distribution has a finite number of extreme points. We can compute the lowest income for each extreme price, analogously to the two good case. Maximizing scale then becomes a linear programming problem with a finite number of inequality constraints. There is an optimal payment that is a vertex of the feasible set, with positive components pinned down by a subset of the hyperplanes defined by the extreme prices.

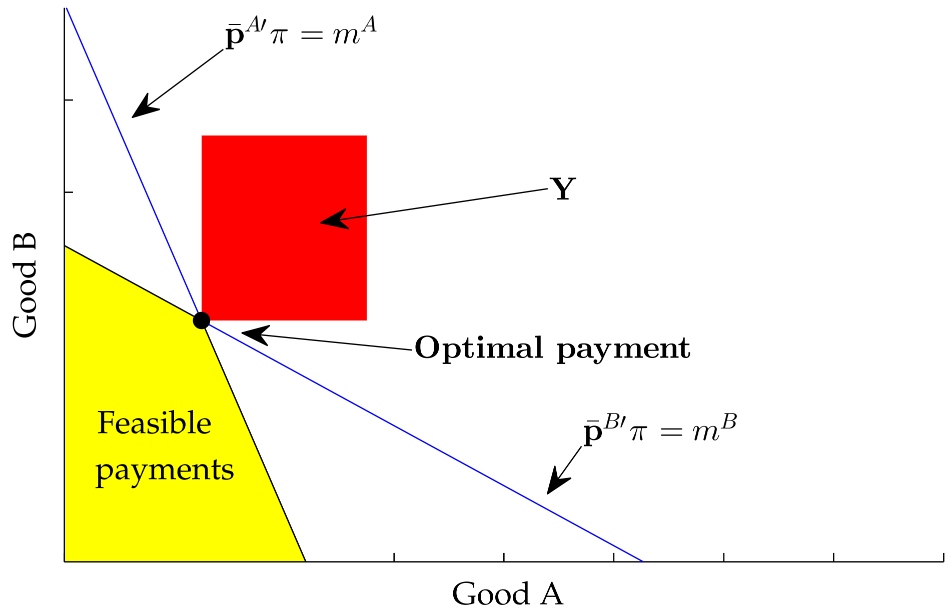

Figure 2: Determination of Optimal Unit of Account with Rectangular Endowment Set $\boldsymbol{Y}$

4 Random Matching and the Optimal Unit of Account in an Entire Economy

The bilateral contracting problem in the previous section shows why the choice of unit of account matters and that the optimal unit of account depends on the type of risk faced by the contracting parties. The results suggest that the distribution of units of account across contracts observed in an economy depends on the distribution of risk. For example, if risk differed a lot across relationships, we might expect to observe a rich cross section of different units of account.

In actual economies, we observe that a dominant unit of account—often, local fiat money—is used in the majority of contracts. In this section we thus explore mechanisms that make risk similar across many contracting relationships. To this end, we embed the bilateral contracting problem into a general equilibrium environment with a large population of agents. We emphasize two features that shape the (endogenous) cross section of risk: gains from forming credit chains and random matching. We show how those features imply the emergence of a dominant unit of account.

In a credit chain, in which agents are both suppliers and customers, bilateral contracting relationships are linked in that one customer's payment is another customer's income. As a result, the nature of income risk can propagate across a chain. We show that this force tends to make the same unit of account optimal throughout a chain. In addition, sequential random matching implies that contracting parties may not know in advance who will join them in a credit chain. It then makes sense to choose a unit of account that is robust to the arrival of many different potential trading partners, and use that unit of account in the majority of transactions.

4.1 Agents, Locations, and Matching

We consider a large population of agents of measure one who differ along two dimensions. First, an agent's location determines his role in the matching process and the potential gains from trade with others. There are three distinct locations with an equal share of agents in each location. Every agent in locations 2 and 3 has a technology to make customized goods one-for-one from labor for a customer he is matched with; we refer to those agents as *artisans*. Agents in location 1 cannot produce customized goods, but receive endowments of market goods; we refer to them as *farmers*.

The geography of the model can be displayed as follows, where the arrow indicates "can produce for:"

Type of Agent:	Farmer	$\longleftarrow$	Artisan	$\longleftarrow$	Artisan
Location:	1		2		3

We can envision the locations to correspond to villages along a highway, with the farmers located at the western end of the highway, and artisans further east in villages 2 and 3, where each artisan in village i can produce for customers in village $i-1$.

The second dimension of heterogeneity is endowment risk. Every farmer receives a random endowment $\boldsymbol{y}$ at date 2, but the distribution can differ across agents. There is a finite number of endowment types $\theta \in \Theta$. For a farmer of type θ, the endowment is drawn from a distribution with compact support $\boldsymbol{Y}(\theta)$ and conditional mean $E(\boldsymbol{y}|\theta)$. Artisans do not receive an endowment: all their income comes from selling customized goods.

At date 0, every artisan is matched with exactly one customer with whom he can engage in bilateral contracting. Matching occurs sequentially from east to west in two stages. In the morning, artisans from locations 2 and 3 meet randomly in pairs. A bilateral contract between these parties specifies a quantity of customized goods x_2 produced by the artisan from location 3 in exchange for a payment of tradable goods $\boldsymbol{\pi}_2$ by the artisan from location 2. At night, agents from locations 1 (farmers) and 2 (artisans) meet to write a contract $(x_1, \boldsymbol{\pi}_1)$.

As a result of the two-stage matching process, every individual ends up as part of a chain of three agents each. At the head of each chain is a farmer from location 1 who has been matched with an artisan from location 2. Each chain also contains an artisan from location 3 who can produce customized goods for the artisan from location 2. In terms of technology and gains from trade, chains only differ in the farmer's endowment type θ. We thus sometimes refer directly to a chain as being of type θ.

A contract for the economy as a whole assigns a bilateral contract to every meeting. Sequential matching restricts the information that bilateral contracts can depend on. In particular, when two agents from locations 2 and 3 meet in the morning, they do not know the endowment type of the farmer who will join their chain at night. The bilateral contract $(x_2, \boldsymbol{\pi}_2)$ must respect agents' information and can therefore not condition on the identity of the farmer in the chain.

4.2 Contracts for the Entire Economy

We denote a contract for the entire economy by $(X, \boldsymbol{\Pi})$, where X and $\boldsymbol{\Pi}$ collect production of customized goods and payments in all meetings, respectively. In particular, given our information structure, a contract consists of a pair $(x_2, \boldsymbol{\pi}_2)$

that describes all bilateral contracts between artisans from locations 2 and 3 determined in the morning, as well as a bilateral contracts $(x_1(\theta), \boldsymbol{\pi}_1(\theta))_{\theta\in\Theta}$ between farmers with different endowment types and artisans from location 2 determined at night.

A key feature of the chain structure is that the feasibility of a payment agreed on between two artisans in the morning depends on a payment that the artisan in location 2 will receive from a customer whom he has yet not met. For a given economywide contract $(X, \mathbf{\Pi})$, we denote by

$$\mathbf{\Pi}_1(\mathbf{\Pi}) = \{\boldsymbol{\pi}_1(\theta), \ \ \theta \in \Theta\}$$

the set of possible payments that an artisan in location 2 can receive from farmer customers in location 1, one of whom he will meet at night.

A contract $(X, \mathbf{\Pi})$ is feasible if every customer can make payments at every stage along the chain, whatever the realization of the price and his income:

$$\boldsymbol{p}'\boldsymbol{\pi}_1(\theta) \leq \boldsymbol{p}'\boldsymbol{y} \quad \text{for all } \boldsymbol{p} \in \boldsymbol{P}, \text{ and } \boldsymbol{y} \in \boldsymbol{Y}(\theta), \tag{11}$$

$$\boldsymbol{p}'\boldsymbol{\pi}_2 \leq \boldsymbol{p}'\boldsymbol{\pi}_1 \quad \text{for all } \boldsymbol{p} \in \boldsymbol{P}, \text{ and } \boldsymbol{\pi}_1 \in \mathbf{\Pi}_1(\mathbf{\Pi}). \tag{12}$$

The first condition (11) says that the farmer's payment is feasible—the constraint is identical to (1) in the bilateral problem from Section 3. The second condition is payment feasibility for the artisan in location 2. It takes the same form as for the farmer, but with the endowment set $\boldsymbol{Y}$ replaced by the set of potential customer payments $\mathbf{\Pi}_1$.

Utility from contracts is as in Section 3, but modified to accommodate that artisans in location 2 can be customers and suppliers at the same time. After matching is complete and the type of the chain θ has been realized, an artisan in location 2 expects utility

$$U_2(X, \mathbf{\Pi}; \theta) = (1+\lambda)\, x_2 + q(\boldsymbol{\pi}_1(\theta)) - q(\boldsymbol{\pi}_2) - x_1(\theta). \tag{13}$$

Here $x_1(\theta)$ is the artisan's labor effort, and the difference in payment scales is the indirect utility the artisan receives from tradable goods he keeps for himself rather than pay to his supplier from location $t+1$.

Agents at the beginning and end of a chain engage in only one bilateral contract. Their utilities are

$$\begin{aligned} U_1(X, \mathbf{\Pi}; \theta) &= (1+\lambda)\, x_1(\theta) - q\left(\boldsymbol{\pi}_1(\theta)\right) + E(\boldsymbol{p}'\boldsymbol{y}|\theta), \\ U_3(X, \mathbf{\Pi}) &= q\left(\boldsymbol{\pi}_2\right) - x_2. \end{aligned} \tag{14}$$

The utility of the farmer at location 1 is the same as that of the customer in the bilateral contract of Section 3; we have only added the farmer type. The utility of the artisan from location 3 is the same as that of the supplier from Section 3. This artisan's utility does not depend on the type of the chain, since the morning contract cannot condition on that type.

4.3 Optimal Payments and the Unit of Account

As in Section 3, we analyze the optimal use of units of account by characterizing the Pareto frontier subject to payment feasibility constraints and individual rationality constraints. A contract is individually rational if every agent is happy to participate in the bilateral contracts assigned to all his matches, conditional on information at the stage of matching. In particular, farmers must be promised utility at least as high as the value of their endowment:

$$U_1(X, \mathbf{\Pi}; \theta) \geq E(\boldsymbol{p}'\boldsymbol{y}|\theta). \tag{15}$$

At the same time, artisans in location 3 must be promised at least as much as their outside option of not working and consuming nothing, that is:

$$U_3(X, \mathbf{\Pi}) \geq 0. \tag{16}$$

Artisans in location 2 must be encouraged to participate in two bilateral contracts. Consider their options in the morning. Like artisans in location 3, they could decide not to work and to consume nothing. In addition, they could also decide to pass on the morning contract and only engage in a bilateral contract with the farmer whom they meet at night. The individual rationality constraint in the morning is therefore given by:

$$E\left(U_2(X, \mathbf{\Pi}; \theta)\right) \geq \max\left\{E\left(q\left(\boldsymbol{\pi}_1\left(\theta\right)\right) - x_1\left(\theta\right)\right), 0\right\}, \tag{17}$$

We take expectations to condition on the artisan's (lack of) information in the morning, when the type of the chain is not yet known. The first term in braces is the expected utility from trading only with the farmer at night.

Consider now the situation of an artisan from location 2 in a night meeting. If he has made a promise in the morning, he has no way to make payment unless he engages in trade with the farmer. In contrast, if he has not made any promise in the morning, then he has the option to pass on the night contract, once he has learned the farmer's type. We thus require the additional individual rationality constraint:

$$q\left(\boldsymbol{\pi}_1\left(\theta\right)\right) - x_1\left(\theta\right) \geq 0 \quad \text{if } \boldsymbol{\pi}_2 = 0. \tag{18}$$

We again solve a planner problem that characterizes constrained Pareto optimal contracts.

Definition 2 (Optimality of Economywide Contracts)**.** *A contract for an entire economy* $(X, \mathbf{\Pi})$ *is (constrained Pareto) optimal if for some utilities* $v_1\left(\theta\right) \geq E(\boldsymbol{p}'\boldsymbol{y}|\theta)$ *and* $v_3 \geq 0$ *it maximizes* $E\left(U_2(X, \mathbf{\Pi}; \theta)\right)$ *subject to* (11) *and* (12) *(i.e., the payments* $\mathbf{\Pi}$ *are feasible);* (15)–(18) *(i.e., the contract is individually rational); and* $U_1(X, \mathbf{\Pi}; \theta) = v_1\left(\theta\right)$ *for all* $\theta \in \Theta$ *and* $U_3(X, \mathbf{\Pi}; \theta) \geq v_3$*.*

In the bilateral contracting problem of Section 3, the optimal payment maximizes the scale of payment subject to feasibility (Lemma 1). Accordingly, we found that the optimal unit of account tailors the payment to the set of potential customer endowments (Propositions 1 and 2). We now show that in the general equilibrium environment here, it is still optimal to maximize scale in every individual meeting. The new element is that the set of potential bundles of tradable goods available to the customer depends on his trading partners. At every stage of matching, the unit of account is thus tailored to the potential bundles, and this determines how the optimal unit of account propagates through the economy.

For an arbitrary set of tradable goods bundles $\tilde{\boldsymbol{\Pi}} \subset \mathbb{R}^2_{\geq 0}$, we denote by $\boldsymbol{\pi}^*(\tilde{\boldsymbol{\Pi}})$ the payment that maximizes the scale of payment $q(\boldsymbol{\pi})$ subject to feasibility, that is, the solution to the bilateral contracting problem set up in Definition 1 with $\boldsymbol{Y} = \tilde{\boldsymbol{\Pi}}$.

Proposition 3 (Optimality of Payments in Economywide Contracts)**.** *A collection of payments $\boldsymbol{\Pi}$ is part of an optimal contract $(X, \boldsymbol{\Pi})$ if and only if it maximizes the scale of payment within each individual meeting, i.e.:*

1. *Payments in meetings between farmers of type θ and artisans from location 2 are $\boldsymbol{\pi}_1(\theta) = \boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))$.*
2. *The payment in all meetings between artisans from locations 2 and 3 is given by $\boldsymbol{\pi}_2 = \boldsymbol{\pi}^*(\boldsymbol{\Pi}_1(\boldsymbol{\Pi}))$.*

Intuitively, an optimal contract has to accomplish two tasks. First, it should maximize gains from trade between parties in every bilateral relationship. We know from Section 3 that this task requires maximizing the scale of payment and the production of customized goods in individual meetings. Second, in the general equilibrium environment here, the contract should use payments and production in any one meeting to facilitate gains from trade in other meetings to the east or west.

The proposition shows that if a payment accomplishes the first task, then it also accomplishes the second. In particular, the optimal payments to artisans in night meetings should maximize scale in those meetings only. The payments should not depend on promises made by those artisans in morning meetings—in fact, they should be as they would be if no morning meetings had taken place. The result builds on Proposition 2: since a payment that maximizes scale also maximizes value at any price, it optimally relaxes feasibility constraints also in meetings further down the credit chain.

By maximizing scale in every meeting, the contract allows maximal transfer of tradable goods through the chain from west to east. It thus allows larger payments for customized goods, which incentivizes more production of these goods.

At the same time, maximizing production of customized goods creates more incentives to work towards the west, which in turn allows larger payments there.

The proposition allows us to recursively trace the optimal unit of account across the economy. In night meetings, we observe as many different units as farmer types. Substituting for $\mathbf{\Pi}_1(\mathbf{\Pi})$ from our knowledge of the night payments, we can write the morning payment as:

$$\boldsymbol{\pi}_2 = \boldsymbol{\pi}^* \left(\left\{\boldsymbol{\pi}^* \left(\boldsymbol{Y}(\theta)\right), \ \ \theta \in \Theta\right\}\right).$$

The unit of account in morning meetings results from applying the function $\boldsymbol{\pi}^*$ twice: first to each farmer's endowment support, and then again to the set of optimal bundles that an artisan from location 2 will possibly receive in night meetings. The dominant unit of account that is used in all morning meetings is thus designed to be robust to all farmer types.

We conclude this section by discussing three extensions. First, one could extend the model to allow for more locations (i.e., additional artisans in location 4, 5, 6 and so on). Applying the logic of Proposition 3 recursively, it would then be optimal to pass along the payment received by artisans in location 2 towards the east along the chain. To accomplish this without further reduction in scale, the same unit of account should be used in all artisan-artisan matches. A dominant unit of account thus emerges from the combination of credit chains (where optimal units of account are passed on) with random matching (so that the same unit of account is used in different chains).

Second, our definition of optimal contracts requires only individual rationality. In an environment with multiple bilateral meetings, it is also attractive to rule out joint deviations by a pair of agents within a meeting. In Appendix B in the online supplement, we define a notion of coalition-proofness for our environment and show that an optimal contract satisfies this additional criterion.

Finally, we have assumed that artisans receive no endowment. We describe an extended environment with risky endowment income for artisans in Appendix C in the online supplement. The main effect is that additional idiosyncratic endowment income makes risk less similar across bilateral contracting relationships.

Much like with less randomness in matching (i.e., certain artisans can only meet a a subset of farmers), there is less coordination on a dominant unit of account throughout the economy. It remains the case that correlation in units of account in the economy is increasing in the length of credit chains and the degree of random matching.

4.4 Examples with Specialized Farmers

We now illustrate our results with concrete examples. The simplest possibility is that there is only one type of farmer who has a certain endowment.

Example 3. *There is one type of farmer, $\theta = A$, where farmer A has a certain endowment of one unit of good A.*

The example features simple income risk in the sense of Proposition 1. Given the proposition, in meetings between a farmer and an artisan from location 2, the optimal payment is given by $\boldsymbol{\pi}_1 = (1, 0)$; that is, the farmer promises the one unit of good A that he is sure to receive, and no units of good B. As a result, good A serves as the sole unit of account. Moreover, in morning meetings between artisans from locations 2 and 3 the same payment $\boldsymbol{\pi}_2 = (1, 0)$ is agreed on. The artisans anticipate that the artisan at location 2 will receive this payment from the contract with the farmer, and hence the scale is maximized by passing on the same payment.

This simple example illustrates the role of credit chains: the same payment and hence the same unit of account is passed on throughout the chain. Notice that the artisans from locations 2 and 3 who meet in the morning neither have an endowment of good A nor do they derive any special enjoyment from it, yet nevertheless they use it as the sole unit of account, because they already know that good A will emerge as the dominant unit of account in their credit chain.

Next, we consider an environment with two types of farmers in a symmetric environment.

Example 4. *There are two types of farmers, $\theta \in \{A, B\}$, where farmer A has an endowment of one unit of good A, and farmer B has an endowment of one unit of good B. Price risk is symmetric in the sense that:*

$$\overline{p} \equiv \max_{\boldsymbol{p} \in \boldsymbol{P}} \left\{ \frac{p^A}{p^B} \right\} = \max_{\boldsymbol{p} \in \boldsymbol{P}} \left\{ \frac{p^B}{p^A} \right\}.$$

Given that each farmer has a certain income, the payment agreed on in meetings between farmers and artisans is once again straightforward: it is given by the farmer's endowment of one unit of good A or B. The choice of payment in morning meetings between artisans from location 2 and 3 is more complex. The artisans know that 2 will meet a farmer at night, but they don't know if it will be a farmer of type A or B. Propositions 2 and 3 imply that the optimal payment $\boldsymbol{\pi}$ should satisfy (9) and (10). In the example, the conditions simplify to:

$$\begin{aligned} \overline{p}\pi^A + \pi^B &= 1, \\ \pi^A + \overline{p}\pi^B &= 1, \end{aligned}$$

resulting in the optimal payment:

$$\boldsymbol{\pi}_2 = \left(\frac{1}{1+\overline{p}}, \frac{1}{1+\overline{p}} \right).$$

Hence, artisan 2 promises an equally weighted bundle of goods A and B, implying that the unit of account is $\boldsymbol{u} = (0.5, 0.5)$. Intuitively, the worst that could happen is that artisan 2 meets farmer A, and then the highest relative price for good B is realized. The optimal bundle is the largest equally weighted bundle that the artisan could afford at this price after receiving a payment of one unit of good A from the farmer. The proposition demonstrates another important feature of optimal units of account: if there is uncertainty over future trading partners, the chosen unit of account should be one that minimizes the variability of the value of the promised payment relative to the income of the possible trading partners.

The next example introduces endowment risk for the two farmers.

Example 5. *There are two types of farmers, $\theta \in \{A, B\}$, where the support of the endowment of type θ is given by an interval $[\underline{y}^\theta, \overline{y}^\theta]$ of good θ, and zero units of the other good. Price risk is symmetric as in Example 4 with a maximum price of $\overline{p}$ of each good relative to the other.*

The farmers' income risk is once again simple, and Proposition 1 applies: the worst-case income is given by $\underline{y}_\theta$ units of good θ regardless of price, and hence farmer θ promises $\underline{y}_\theta$ units of good θ and zero units of the other good.

Now consider the morning contract between artisans 2 and 3. Artisan 2 knows that later on, he will either meet A and receive a payment of at least $\underline{y}^A$ of good A, or he will meet B and receive a payment of at least $\underline{y}^B$ of good B. Let $\underline{y}^B \leq \underline{y}^A$. From Propositions 2 and 3, the optimal payment $\boldsymbol{\pi}$ should satisfy:

$$\begin{aligned}
\overline{p}\pi^A + \pi^B &= \underline{y}^B, \\
\pi^A + \overline{p}\pi^B &= \min\left\{\underline{y}^A, \overline{p}\underline{y}^B\right\}.
\end{aligned}$$

If $\overline{p}\underline{y}^B \leq \underline{y}^A$, the artisan at location 2 faces simple income risk given by $\boldsymbol{y}_0 = (0, \underline{y}^B)$. Hence, in this case the optimal payment is equal to this worst income realization:

$$\boldsymbol{\pi}_2 = \left(0,\ \underline{y}^B\right),$$

and good B is the sole unit of account in these meetings. Given that the same unit of account is used in evening meetings of artisan 2 with farmer B, good B becomes the dominant unit of account: it is used in three out of every four meetings in the economy. If instead we have $\overline{p}\underline{y}^B > \underline{y}^A$, the optimal payment is given by:

$$\boldsymbol{\pi}_2 = \left(\frac{\overline{p}\underline{y}^B - \underline{y}^A}{\overline{p}^2 - 1},\ \frac{\overline{p}\underline{y}^A - \underline{y}^B}{\overline{p}^2 - 1}\right).$$

Hence, the optimal payment is a bundle, but the bundle places higher weight on the good with the lower minimum endowment, so as to hedge against the possibility of meeting a farmer who produces this good. The scale of the payment is declining in price risk, i.e., the maximum relative price $\overline{p}$.

Finally, we consider an example with asymmetric price risk.

Example 6. *There are two types of farmers, $\theta \in \{A, B\}$, where the support of the endowment of each type θ is given by the same interval $[\underline{y}, \overline{y}]$ of good θ, and zero units of the other good. Price risk is asymmetric with a maximum relative price $\overline{p}^\theta$ of each good relative to the other, with $\overline{p}^A \neq \overline{p}^B$.*

Given that the relationship of farmer and artisan is still characterized by simple income risk, we proceed directly to the optimal payment between artisans 2 and 3. The conditions for optimality are:

$$\begin{aligned}\overline{p}^A \pi^A + \pi^B &= \underline{y}, \\ \pi^A + \overline{p}^B \pi^B &= \underline{y},\end{aligned}$$

and the optimal payment is:

$$\boldsymbol{\pi}_2 = \left(\frac{\left(\overline{p}^B - 1\right) \underline{y}}{\overline{p}^A \overline{p}^B - 1}, \frac{\left(\overline{p}^A - 1\right) \underline{y}}{\overline{p}^A \overline{p}^B - 1} \right).$$

Hence, the optimal bundle places more weight on the good with the lower price risk (lower $\overline{p}^\theta$). The example shows that for given endowment risk, the optimal unit of account will place more weight on goods that are more stable in value, and hence lead to less uncertainty about the value of promised payments.

4.5 Optimal Production and the Pareto Frontier

The optimal payment and hence the optimal unit of account are independent of where on the Pareto frontier the optimal contract is located: it is always beneficial to maximize the scale of payments in meetings. The weight placed by a social planner on different agent types only affects the customized goods that those agents receive or produce. The next proposition shows which utility promises to farmers and artisans in location 3 are feasible and how the optimal production of customized goods depends on those promises.

Proposition 4 (Optimality of Production in Economywide Contracts)**.** *An optimal contract can promise farmers and artisans in location 3 any utilities $v_1(\theta)$ and v_3 that* *satisfy:*

$$\frac{\lambda}{1+\lambda} q_2 \left(\boldsymbol{\pi}_2 \right) \geq v_3, \tag{19}$$

$$\frac{\lambda}{1+\lambda} q_2 \left(\boldsymbol{\pi}_2 \right) + \frac{\lambda}{(1+\lambda)^2} E \left(q \left(\boldsymbol{\pi}_1 \left(\theta \right) \right) \right) \geq v_3 + \frac{1}{(1+\lambda)^2} E(v_1 \left(\theta \right) - \boldsymbol{p}' \boldsymbol{y}). \tag{20}$$

The optimal production of customized goods is:

$$x_1 \left(\theta \right) = \left(1+\lambda \right)^{-1} \left(q \left(\boldsymbol{\pi}_1 \left(\theta \right) \right) + v_1 \left(\theta \right) - E(\boldsymbol{p}' \boldsymbol{y} | \theta) \right), \tag{21}$$

$$x_2 = q_2 \left(\boldsymbol{\pi}_2 \right) - v_3. \tag{22}$$

Condition (20) is equivalent to the requirement that $E \left(U_2(X, \boldsymbol{\pi}; \theta) \right) \geq 0$. The condition allows us to trace out the entire Pareto frontier by relating the utility of artisans in location 2 to the utility promised to other types. Up to a scale factor, the utility of artisans in location 2 is equal to the weighted surplus generated by production of customized goods (the left-hand side) less the weighted utility promises to farmers and artisans in location 3 (the right hand side).

The weights multiplying q_1 and q_2 on the left-hand side indicate the marginal value of payment capacity in night and morning meetings, respectively. They answer the question: how much would total surplus increase, if customers in a given meeting were able to pay one unit more (say, because of a change in the environment). The marginal value of payment capacity is λ: if the customer (the farmer) can pay one more unit, then the supplier (the artisan from location 2) can produce $1+\lambda$ more units of the customized good.

In contrast, the marginal value of payment capacity in a morning meeting is $\lambda+\lambda^2$. There are λ units of surplus realized in the morning meeting itself: as above, if the customer can pay one more unit, then the supplier (the artisan in location 3) can produce $1+\lambda$ more units of the customized good. The additional λ^2 units of surplus are due to the fact that surplus in morning meetings helps generate surplus in night meetings.

The key effect here is that the customized good arranged in the morning meeting serves as payment to the artisan in location 2. This is valuable, since payment capacity in night meetings is limited. Specifically, since we have seen that the marginal value of payment capacity in night meetings is λ, a unit of payment capacity in a morning meeting contributes λ^2 units of surplus generated in night meetings.

By a similar argument, promising one unit of utility to a farmer is cheaper for the social planner than promising that unit to an artisan in location 3. Any rents obtained by the latter artisan require shifting resources through the chain for payment without having the artisan work. To reflect the lower cost, the farmer's utility promise—net of the farmer's outside option of eating his endowment—receives lower weight $(1+\lambda)^{-2}$.

Condition (19) resembles the bound for the supplier's maximal utility in Section 3. The promise to an artisan in location 3 is limited by the ability to pay of the customer who must consume the artisan's production, measured by $q_2(\boldsymbol{\pi}_2)$. Artisans in location 3 cannot receive more rents than if they were in a bilateral relationship with the farmer who receives the highest income. It is important here that we allow artisans in location 2 to choose not to enter the morning contract. If this was prohibited (i.e., the right hand side of constraint (17) was zero), promises only would have to satisfy (20).

4.6 Decentralizing the Optimal Contract

So far, we have focused on constrained Pareto optimal allocations. We have characterized those allocations as solutions to a social planner problem. Implicitly, the outcome can be thought of as the planner making a contract proposal to all agents who meet, with the agents being able to either accept or reject the proposal. In this section, we show that optimal economywide contracts arise as equilibria of another game that involves decentralized interactions between agents in the economy.

The game respects the trading constraints and information structure of the environment. For strategies and payoffs to be well defined, we need to take a stand on two issues. The first is negotiation within individual meetings. Here we assume that agents engage in Nash bargaining with a particular set of weights. In particular, we show below how to select bargaining weights in different types of meetings in order to decentralize different points on the Pareto frontier.

The second issue is how contracts are enforced, especially when negotiation in individual meetings results in infeasible chains of payment. We make two assumptions. First, agents have access to a court system that grants limited liability: if an agent has promised a payment that he cannot make, then he can declare bankruptcy. An agent who declares bankruptcy does not work or consume and hence receives a payoff of zero. This option is relevant for artisans in location 2 who take part in two meetings.

The assumption of limited liability is useful because it implies a simple formulation for the outside options that underlie the Nash bargaining between pairs of agents. Without limited liability, we would have to make further assumptions on what happens to agents who have made a promise, and then encounter an agent who is unwilling to provide them with a payment needed to meet this promise. Limited liability, however, leads to an additional issue that needs to be dealt with, namely the possibility that it could lead to strategic default by artisans. Our second assumption is that there is a regulatory agency which monitors promises and sets an upper limit on payment promises. In particular, we bound the value of the payment promise at the price vector that minimizes that value—a kind of stress test on borrowers. For simplicity, we impose this bound only on artisans in location 2. While these assumptions are special, they deliver a concept of equilibrium such that all constrained Pareto optima can be implemented as equilibria of the game.

Limited liability also implies that that the decentralization result does not follow directly from the Pareto-efficiency property of Nash bargaining together with the fact that the optimal contract is coalition proof (see Appendix B). In the analysis of coalition proofness, we assume that agents can only deviate to feasible contracts, which amounts to allowing the planner to inflict arbitrarily large default costs on agents who promise something infeasible. Here we consider a decentralization under weaker and more appealing assumptions on contract enforcement that lead to a simple formulation of the outside options underlying Nash bargaining.

Definition 3 (Equilibrium with Bargaining)**.** *An equilibrium of an economy with bargaining weights μ_3 and $(\mu_1(\theta))$ and a bound $\bar{m}_2$ on the value promised by artisan 2 consists of a bilateral contract in morning meetings $(x_2, \boldsymbol{\pi}_2)$ as well as a bilateral contracts in night meetings $(x_1(\theta), \boldsymbol{\pi}_1(\theta))$ for every history of bilateral contracts determined in morning meetings such that:*

i) Bilateral contracts in morning meetings are determined by Nash bargaining with weight μ_3 on the artisan from location 3.

ii) Bilateral contracts in night meetings with farmer type θ are determined by Nash bargaining with weight $\mu_1(\theta)$ on the farmer.

iii) If payments negotiated along a chain are infeasible for some price, the artisan in location 2 neither works nor consumes.

We can now establish our decentralization result.

Proposition 5 (Decentralization of Optimal Economywide Contracts)**.** *For every collection of possible utility promises v_3 and $(v_1(\theta))_{\theta\in\Theta}$, there is a collection of bargaining weights μ_3 and $(\mu_1(\theta))_{\theta\in\Theta}$ and a bound $\bar{m}_2$ such that the optimal contract given the utility promises is an equilibrium of the economy with those bargaining weights.*

The two assumptions on enforcement ensure that Nash bargaining can be used to apportion the overall surplus across the three types of agents for all constrained-optimal economywide contracts. Limited liability implies that the farmer and the artisan from location 2 can split surplus generated by production $x_1(\theta)$ as well as surplus generated by production x_2. The regulatory agency ensures that the artisan in location 2 cannot game the system given that limited liability is in place. Suppose, for example, that there are two farmer types, one of whom produces at much higher scale than the other. It might then make sense for the artisans in a morning meeting to negotiate high production and a payment that can only be met if the high-scale farmer joins their chain at night. They would then default on the low-scale farmer who cannot share in surplus.

5 Government Debt and the Choice of an Optimal Unit of Account

The preceding analysis has shown how a dominant unit of account leads to better allocations in economies characterized by relative price risk, credit chains, and uncertainty about future trading partners. However, the optimal unit of account generally turns out to be a bundle of goods. In actual economies, in contrast, the dominant unit of account usually consists of government-issued money, such as euros or dollars. In this section, we explore how our theory can be extended to account for the prominent role of government paper in real-world units of account.

Our explanation for the use of government-issued money as a common unit of account builds on the results in Sections 3 and 4: the unit of account should reflect the income risk of borrowers. Government money enters balance sheets through money-denominated (i.e., nominal) assets. The most important example of such an asset is nominal government debt. Issuing debt in nominal terms has clear advantages for the government; nominal debt is implicitly state-contingent (through the government's control of inflation) and can therefore provide insurance for future government spending shocks (this role of nominal government debt in a stochastic macroeconomic environment was first pointed out by Bohn 1988). If a lot of government debt is in circulation, private agents derive more of their income in nominal terms (through interest payments and principal repayment on government bonds), which makes money more attractive as a unit of account for private transactions, too.

To articulate this mechanism within the framework of our model, we build on the symmetric Example 4 above. Hence, there are two types of farmers, $\theta \in \{A, B\}$, where farmer A has an endowment of one unit of good A, and farmer B has an endowment of one unit of good B. Price risk is symmetric with the maximum price of each good relative to the other given by $\overline{p} > 0$. Let $\underline{p} = 1/\overline{p}$ denote the minimum relative price of each good. Into this environment we introduce a new actor, the government. To focus on government debt as the optimal unit of account, the only role of the government is to issue IOUs (government debt in the form of pieces of paper) and to repay them later on. Specifically, at date 0 (i.e., before price uncertainty has been realized) the government acquires a claim on g units of each farmer's output, and in exchange issues g units of government IOUs to each farmer. A unit of IOU is defined as a claim on one unit of (stochastic) government revenue T, where the expected value of revenue T in the spot market is $E_0(T) = 1$. Government IOUs are traded in the centralized spot market, and can thus serve as a unit of account.[5]

The price of government IOUs is volatile. Specifically, in period 1, after contracts are written but before the spot market opens, news about government revenue arrives. Since agents are risk neutral, the expected value of revenue $E_1(T)$ pins down the price of IOUs in the spot market: $p^{IOU} = E_1(T)$. To ensure that IOU prices are symmetric with regards to goods A and B, we maintain the following assumption:

Assumption 2 (Symmetric IOU prices relative to goods A and B)**.** *The support of the price distribution for IOUs is given by:*

$$p^{IOU} \in \left[\hat{p}(\boldsymbol{p})\, \underline{p}^{IOU}, \hat{p}(\boldsymbol{p})\, \overline{p}^{IOU}\right],$$

where:

$$\hat{p}(\boldsymbol{p}) = \left(\left(\frac{p^A/p^B - \underline{p}}{\overline{p} - \underline{p}} \right) p^A + \left(\frac{\overline{p} - p^A/p^B}{\overline{p} - \underline{p}} \right) p^B \right)$$

and:

$$\underline{p}^{IOU} \leq \frac{\underline{p} + 1}{2}. \tag{23}$$

Here $\underline{p}^{IOU}$ and $\overline{p}^{IOU}$ give the lower and upper bound of the price of IOUs relative to the expensive farm good at the two extremes of the relative price of good A and good B. In addition, condition (23) states that IOU prices are sufficiently

[5] In physical terms, government revenue T could consist either of bundles of goods A and B or of other goods that are traded in the centralized market. For the agents in our model, all that matters is the value of IOUs in the centralized market. Likewise, for determining the optimal unit of account it is immaterial *why* IOUs are valued. We choose to anchor the value of IOUs through claims on government revenue for simplicity, but for a given distribution of the market value of IOUs, alternative ways of founding the value of IOUs (such as models of valued fiat money) would give the same results.

variable such that the relative price of IOUs can be lower than that of a equally weighted bundle of the farm goods.

In terms of choosing units of account, the central new feature is that a farmer of type i, rather than deriving all income from farm good i, now derives income partially from the farm good and partially from the government IOU. Since the expected tax revenue (and hence the expected value of an IOU) equals one, this does not change the farmer's expected income. However, the presence of government IOUs does change the optimal unit of account in the economy. As before, in meetings between farmers and artisans and location 2 the optimal payment consists of the entire endowment of the farmer, in this case a combination of the farm good and government IOUs. The question is what should serve as unit of account in morning meetings between artisans from locations 2 and 3. The following proposition characterizes the optimal unit of account in the economy with circulating government paper.

Proposition 6. *Let the distributions of farm-good and IOU prices satisfy Assumption 2. Government IOUs in circulation can serve as a unit of account, so that payment promises from artisans at location 2 to artisans at location 3 are given by:*

$$\boldsymbol{\pi}_2 = (\pi^{IOU},\ \pi^A,\ \pi^B)' = q\ (u^{IOU},\ u^A,\ u^B)'$$

with $u^{IOU} + u^A + u^B = 1$, $u^{IOU}, u^A, u^B \geq 0$. *We then have:*

1. *If* $\overline{p}^{IOU} > \frac{\underline{p}+1}{2}$, *the optimal unit of account in all artisan-artisan matches is given by:*

$$u^{IOU} = \frac{g}{g + (1-g)\frac{2\underline{p}}{\underline{p}+1}} \equiv \tilde{u}^{IOU}, \tag{24}$$

$$u^A = u^B = \frac{1 - u^{IOU}}{2}. \tag{25}$$

2. *If* $\overline{p}^{IOU} \leq \frac{\underline{p}+1}{2}$, *the optimal unit of account in all artisan-artisan matches is given by government IOUs, i.e., we have* $u^{IOU} = 1$ *and payments can be written as:*

$$\boldsymbol{\pi}_2 = q\ (1,\ 0,\ 0).'$$

That is, if the price of IOUs is volatile, the optimal unit of account is a combination of IOUs and an equally-weighted bundle of farm goods, with the weight on IOUs increasing in the amount g of IOUs in circulation. If the price of government paper is relatively stable, IOUs are the sole unit of account.

To gain intuition, consider the first case in the proposition, which applies when the price bounds for IOUs are relatively wide, i.e., the value of government debt is volatile. In this case it is optimal to hedge against the exposure to IOUs by making the weight of IOUs in the unit of account increasing in the quantity g of IOUs in circulation. The intuition for using a bundle of both farm goods and IOUs is as in Example 4 above; the objective is to choose a unit of account for making promises that has a low value in the "worst case" price realization; by including all goods in the bundle, the "cheap" good is always included, which moderates variation in the value of the promise relative to income received. The weight of IOUs in the optimal bundle is increasing in g to align the value of the unit of account with the value of income received. In farmer-artisan matches, the farmer promises g units of IOUs and $1-g$ units of farm goods, so that the share of IOUs in artisans' income is increasing in g also.

The second case of the proposition shows conditions under which the optimal unit of account consists solely of IOUs rather than a bundle of goods. Consider the risk that arises from receiving either farm good A or B as in Example 4 above. The optimal unit of account should be chosen to have the lowest possible value at the extremes of the price distribution. An equally weighted bundle of A and B has value $(\underline{p}+1)/2$ relative to the expensive good when the cheap good is at its lowest relative price. What would be better is to use a unit of account that has an even lower value when either one of the farm goods reaches its lowest relative price. If the condition in the second case of Proposition 6 is satisfied, IOUs are such a good, and are therefore used as the sole unit of account. Intuitively, the condition can be understood in terms of the volatility of the price of IOUs. If the upper bound for the relative price of IOUs $\overline{p}^{IOU}$ is low, the value of IOUs is relatively stable, which makes IOUs a better unit of account than a bundle of goods with more volatile value.

It might be the case that condition (23) does not hold, in which case it is possible that IOUs will not enter the optimal unit of account. However, if we generalize the model to allow for many farm goods, it is plausible that (23) will hold. The issue here is the value of an equally-weighted bundle of farm goods in the worst-case scenario in terms of meeting the payment feasibility constraint. In the worst-case scenario, the relative price of the farm good received will be at the minimum while the relative price of all other farm goods will be at the maximum, suggesting that the relative price of an equally weighted bundle of farm goods will be high.

To translate these results into more familiar terms, we can refer to an IOU as a "euro." If the euro is the unit of account, the consumer price index is CPI $=$ $(p^{IOU})^{-1}$. A high volatility of the price of euros then translates into a volatile CPI, i.e., volatile inflation. The worst-case scenario that drives the choice of the optimal unit of account is one of a high p^{IOU} and hence a low CPI. Intuitively, when the euro is the unit of account, artisan 2 promises a fixed number of euros to artisan 3. If now realized inflation is low or negative (deflation), the real value of that euro-denominated promise is high, possibly leading to a binding payment feasibility constraint (as in Fisher's debt-deflation theory). If inflation becomes too volatile, the euro ultimately is no longer the optimal unit of account. This is akin to the dollarization of an economy when the local currency becomes overly volatile, and alternative units of account (such as foreign currency) start to be used.[6] A sudden spike in p^{IOU} could also be generated by government-mandated changes in the value of the unit of account, as in the historical sudden deflation episodes analyzed by Velde (2009).

The analysis provides a rationale for why government-issued money often, but not always, arises as the dominant unit of account in an economy. In addition to the formal arguments provided by Proposition 6, another force that favors using government paper as the unit of account is that using bundles as a unit of account is more complicated that using a single unit of account. Hence, if there are contracting frictions that favor making promises in terms of a single unit, using government paper may be optimal even under the conditions where Proposition 6 would dictate using a bundle of IOUs and farm goods.

[6]See Neumeyer (1998) for a general-equilibrium analysis of the breakdown of trade in nominal assets when inflation risk becomes too high.

6 Optimal Currency Areas

An important practical issue regarding the use of units of account is the question of optimal currency areas, i.e., under what conditions do multiple regions or countries have an incentive to adopt a common unit of account? To address this question, we now modify our model to allow for variation in the intensity of linkages between regions. So far, we have assumed that matching at each link is entirely random; meetings between an artisan from location 2 and any given farmer are equally likely. We now add a second dimension of geography: agents live in two different regions, and are more likely to be matched to people within their region than to those outside. In this setting, a tension arises between adopting a "global" unit versus adopting several "regional" units of account that are more suited to local conditions.[7] The analysis therefore leads to a theory of optimal currency areas, where the optimality of a common unit of account depends on the degree of specialization across countries and on the intensity of cross-border links.

We once again build on the symmetric Example 4 with two types of farmers, $\theta \in \{A, B\}$, where farmer A has an endowment of one unit of good A, and farmer B has an endowment of one unit of good B. Price risk is symmetric with the maximum price of each good relative to the other given by $\overline{p} > 1$. Different from Example 4, assume that artisans are located in two regions, A and B, corresponding to farmers of type A and B. In the morning, artisans at locations 2 and 3 meet within their region, i.e., location-2 artisans in region A meet location-3 artisans in region A, and the same for region B. At night, an artisan from location 2 in region A meets a farmer from region A with probability $1 - \alpha$, where $0 \leq \alpha \leq 0.5$. With probability α he meets a farmer from region B. Similarly, an artisan from location 2 in region B meets a farmer from his own region with probability $1 - \alpha$.

Without further assumptions, as long as $\alpha > 0$ the optimal unit of account is still as characterized in the discussion of Example 4, because only the possibility (rather than the probability) of meeting a "foreigner" matters. Here we enrich the setting by adding the possibility that an artisan who is mismatched can pay a

[7] Related issues arise in the search-theoretic models of Matsuyama, Kiyotaki, and Matsui (1993) and Wright and Trejos (2001), in which money is used as a medium of exchange.

"rematching cost" $\tau > 0$ (in utils) to switch to the other region. Hence, in contracting there is now a choice between writing contracts so that they are compatible with meeting trading partners from either region, or to write contracts tailored to trading partners from a particular region, which includes the necessity of paying the rematching cost if initially mismatched.

We can now show that the optimality of adopting a common unit of account for both regions depends on the intensity of cross-border trade α. We state the result for a specific distribution of welfare weights/bargaining power across agents; other welfare weights would affect the threshold for α above which a unified currency area is optimal, but not change the basic result.

Proposition 7. *Consider the allocation where all bargaining power rests with artisans in location 2, i.e., $\mu_1(\theta) = \mu_3 = 0$. If $0 < \alpha \leq \tilde{\alpha}$, where*

$$\tilde{\alpha} = \frac{(1+\lambda)(\overline{p}-1)}{\tau(1+\overline{p})}, \tag{26}$$

optimal contracts are as in Example 3 (separate units of account in regions A and B), and mismatched artisans pay the cost τ to match with someone in their own region. If $\alpha > \tilde{\alpha}$, the solution to the planning problem is as characterized in the discussion of Example 4, that is, an equally weighted bundle of goods A and B serves as common unit of account in all morning meetings in both regions, and all agents contract with their initial match rather than paying the transport cost (that is, the economy has a common unit of account or a "currency union").

Hence, we find that the benefits of a currency union increase in the intensity of cross-border trade α. Moreover, the benefits of a union are also higher (i.e., the threshold for α is lower) if there is less relative price risk across the regions, i.e., if $\overline{p}$ is closer to one. This result mirrors findings in traditional analyses of optimal currency areas (e.g., Alesina and Barro 2002) that the benefits of a currency union are higher if the members experience correlated shocks. Notice, however, that the mechanism is entirely different: whereas traditional models emphasize the potential benefits of independent macroeconomic policy, in our theory the tradeoff in adopting a currency union involves risk exposures in contracts among private parties. Another distinct implication of our theory is that the benefits of a currency union are increasing also in the length of credit chains. Consider a simple extension of the setup in which the probability α of meeting someone from the other region applies, separately, to each level of the chain of artisans. This implies that as the number of locations increases, for a given α there is an increase in the probability that each credit chain contains at least one agent from the other region. Thus, for fixed α the benefits of a currency union are increasing in the number of locations.

7 Conclusions

Our goal in this paper was to provide a rationale for why a dominant unit of account might emerge and what it should look like. We start from a bilateral contracting problem with relative price risk and a cost of breaking promises. The unit of account for promises then matters and should track borrower income. In a general equilibrium environment, a dominant unit emerges when there are gains of trade along credit chains and random matching with business partners. This dominant unit no longer tracks individual borrower income, since it must address both price risk and matching risk. It should therefore be stable in value relative to the income of many borrowers.

Our analysis suggests a number of interesting avenues for future research. One is to study the choice of unit of account in more specific contexts. We have considered a fairly abstract environment in order to isolate forces that are relevant in many settings where price and matching risk are present. In particular applications—such as the choice of currency for invoicing in international trade or the choice of debt denomination in international banking—additional forces may emerge. For example, if borrowing requires collateral, the exposure of collateral to relative price risk becomes an issue. If some gains from trade are due to risk sharing, then relative risk attitudes matter. Finally, in some trading net-works the balance-sheet risks that are central to our theory are highly concentrated among intermediaries such as banks.[8]

A second direction for further theoretical research is to explore dynamics. We have considered a static setup and derived comparative statics predictions for the optimal unit of account. At the same time, the mechanism we emphasize is relevant also in a dynamic setup where shocks alter the properties of potential units of accounts. For example, in the presence of many bilateral long term contracts, it may be difficult for society to quickly change the dominant unit of account. The ability to switch units is relevant for understanding the response of an economy that uses government debt as its unit of account to a new policy regime with less stable value. At what point do agents start indexing contracts or use a foreign currency such as the dollar?

Finally, an important challenge for future research is to more formally confront the implications of the theory with empirical evidence. One approach here is to exploit time series variation, as we loosely did in our discussion of historical evidence above. Over long periods of time, it is plausible that the unit of account adapts to changes in the environment, such as the intensity of international trade, the size of the government, or the ability of the government to commit to a stable value of debt. At the same time, it is interesting to consider predictions on the cross section of contracts and individual balance sheets that can be matched to new micro data sets on credit markets and international trade relationships.

A Proofs for Propositions

Proof of Proposition 1: The proof for Part 1 is contained in the main text. Now consider Part 2 regarding uniqueness. Suppose there is a payment $\tilde{\boldsymbol{\pi}} \neq \boldsymbol{y}_0$ that yields the same payoff, $E(\boldsymbol{p}'\tilde{\boldsymbol{\pi}}) = E(\boldsymbol{p}'\boldsymbol{y}_0)$. Given that the promised payment is different from $\boldsymbol{y}_0$ yet yields the same utility, there must be one good i for which $\tilde{\boldsymbol{\pi}}$ promises strictly more than y_0^i, whereas for the other good j, $\tilde{\boldsymbol{\pi}}$ promises strictly

[8]Our analysis has also abstracted from possible connections between the roles of money as a unit of account and as a medium of exchange. Indeed, in our theory all units of account are traded in Walrasian spot markets. While this is useful to analyze the demand for pure accounting currencies, future research could also explore settings where a connection between the different functions is natural, such as the design of clearing and settlement systems for asset market trade.

less than y_0^j. Now consider a price vector $\tilde{\boldsymbol{p}}$ that has $\tilde{p}^i = E(p^i) - \epsilon_1 = 1 - \epsilon_1$ and $\tilde{p}^j = E(p^i) + \epsilon_1 = 1 + \epsilon_2$, with $\epsilon_1, \epsilon_2 > 0$ and $\tilde{\boldsymbol{p}}'\boldsymbol{y}_0 = E(\boldsymbol{p}'\boldsymbol{y}_0)$. Such a price vector exists in $\boldsymbol{P}$ for small enough ϵ_1, ϵ_2 because of the assumed price risk in both goods and the convexity of $\boldsymbol{P}$. Now if $\tilde{\boldsymbol{p}}$ and $\boldsymbol{y}_0$ are realized, we have $\tilde{\boldsymbol{p}}'\tilde{\boldsymbol{\pi}} > \tilde{\boldsymbol{p}}'\boldsymbol{y}_0$. Hence, any such $\tilde{\boldsymbol{\pi}}$ violates feasibility (1), implying that setting $\boldsymbol{\pi} = \boldsymbol{y}_0$ is the unique optimum. □

Proof of Proposition 2: We show first that a bundle is feasible if and only if it satisfies $\bar{\boldsymbol{p}}^{i\prime}\boldsymbol{\pi} \leq m^i$ for $i \in \{A, B\}$, The inequalities are necessary for feasibility, because there exist $\boldsymbol{y} \in \boldsymbol{Y}$ that attain incomes m^A and m^B, and the optimal payment $\boldsymbol{\pi}$ has to satisfy (1) for any $\boldsymbol{y} \in \boldsymbol{Y}$, $\boldsymbol{p} \in \boldsymbol{P}$.

We also need to show that the inequalities are sufficient for feasibility. Consider a bundle that satisfies the inequalities and consider any other endowment $\boldsymbol{y} \in \boldsymbol{Y}$. From the definition of m^A and m^B, we know that $\bar{\boldsymbol{p}}^{A\prime}\boldsymbol{\pi} \leq \bar{\boldsymbol{p}}^{A\prime}\boldsymbol{y}$ and $\bar{\boldsymbol{p}}^{B\prime}\boldsymbol{\pi} \leq \bar{\boldsymbol{p}}^{B\prime}\boldsymbol{y}$.

Consider any other price $\boldsymbol{p} \in \boldsymbol{P}$. Suppose $\pi^A \geq y^A$. Since $\bar{\boldsymbol{p}}^A$ maximizes the relative price of good A and $\bar{\boldsymbol{p}}^{A\prime}\boldsymbol{\pi} \leq \bar{\boldsymbol{p}}^{A\prime}\boldsymbol{y}$, we must have $\boldsymbol{p}'\boldsymbol{\pi} \leq \boldsymbol{p}'\boldsymbol{y}$. Now suppose instead that $\pi^A < y^A$. Since $\bar{\boldsymbol{p}}^B$ minimizes the relative price of good A and $\bar{\boldsymbol{p}}^{B\prime}\boldsymbol{\pi} \leq \bar{\boldsymbol{p}}^{B\prime}\boldsymbol{y}$, we must have $\boldsymbol{p}'\boldsymbol{\pi} \leq \boldsymbol{p}'\boldsymbol{y}$.

Now consider the equations (9) and (10). If there is price risk as stated in the proposition, the matrix on the left hand side is nonsingular, so there is a unique solution $\boldsymbol{\pi}_0$. The solution is feasible since it satisfies the inequalities. To show that it is optimal, consider any other feasible $\boldsymbol{\pi} \neq \boldsymbol{\pi}_0$. By construction of $\boldsymbol{\pi}_0$, we know that $\bar{\boldsymbol{p}}^{A\prime}\boldsymbol{\pi} \leq \bar{\boldsymbol{p}}^{A\prime}\boldsymbol{\pi}_0$ and $\bar{\boldsymbol{p}}^{B\prime}\boldsymbol{\pi} \leq \bar{\boldsymbol{p}}^{B\prime}\boldsymbol{\pi}_0$.

Consider any other price $\boldsymbol{p} \in \boldsymbol{P}$ with $\boldsymbol{p} \neq \bar{\boldsymbol{p}}^A$ and $\boldsymbol{p} \neq \bar{\boldsymbol{p}}^B$. Suppose $\pi^A \geq \pi_0^A$. Since $\bar{\boldsymbol{p}}^A$ is the unique maximizer of the relative price of good A, we must have $\boldsymbol{p}'\boldsymbol{\pi} < \boldsymbol{p}'\boldsymbol{\pi}_0$. Suppose instead $\pi^A < y^A$. Since $\bar{\boldsymbol{p}}^B$ is the unique minimizer of the relative price of good B and $\bar{\boldsymbol{p}}^{B\prime}\boldsymbol{\pi} \leq \bar{\boldsymbol{p}}^{B\prime}\boldsymbol{\pi}_0$, we must have $\boldsymbol{p}'\boldsymbol{\pi} < \boldsymbol{p}'\boldsymbol{\pi}_0$. We conclude that the bundle $\boldsymbol{\pi}_0$ is the the unique bundle that maximizes the value $\boldsymbol{p}'\boldsymbol{\pi}$ subject to (1). □

Proof of Proposition 3: (Necessity) We show first than an optimal contract maximizes the scale of payment within each individual meeting. If there is no optimal contract for given $v_1(\theta)$ and v_3, then there is nothing to show. Suppose then that an optimal contract exists.

We want to show that an optimal contract $(X, \boldsymbol{\Pi})$ satisfies $\boldsymbol{\pi}_1(\theta) = \boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))$ for all $\theta \in \Theta$. Suppose that $\boldsymbol{\pi}_1(\theta) \neq \boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))$ for some θ. Since $\boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))$ maximizes scale for the endowment set $\boldsymbol{Y}(\theta)$, we know that $\Delta q = q(\boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))) - q(\boldsymbol{\pi}_1(\theta)) > 0$. Consider an alternative contract that makes farmer θ pay $\boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))$ and leaves all other payments unchanged.

From (14), we can reduce $x_1(\theta)$ by $\Delta q(1+\lambda)^{-1}$ and leave the utility of farmer θ unchanged. At the same time, we strictly increase the expected utility of the artisan in location 2, which is our objective function.

The alternative contract is feasible. Indeed, it satisfies farmer feasibility constraints by construction. Moreover, Proposition 2 says that the payment $\boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))$ maximizes the value $\boldsymbol{p}'\boldsymbol{\pi}$ among all feasible bundles for *any* price $\boldsymbol{p} \in \boldsymbol{P}$. It follows that the feasibility constraint of the artisan in location 2 still allows the original payment $\boldsymbol{\pi}_2$.

Since the alternative contract does not change the payment of artisan 2, we can retain the same production x_2 and leave the individual rationality constraint of artisan 3 unchanged.

Given the original production x_2, the alternative contract also satisfies the individual rationality constraints of artisan 2. Indeed, From (13), a contract satisfies the individual rationality constraint (17) of artisan 2 if and only if we have $E(U_2(X,\boldsymbol{\Pi};\theta)) \geq 0$ and $(1+\lambda)x_2 - q(\boldsymbol{\pi}_2) \geq 0$.The expected utility of artisan 2 strictly increases so $E(U_2(X,\boldsymbol{\Pi};\theta)) > 0$. The second condition continues to hold since $\boldsymbol{\pi}_2$ and x_2 are unchanged from the original contract which was optimal. Finally, since the alternative contract changes the component of artisan 2 utility that is due to trade with the farmer, (18) also continues to hold.

We can also retain production $x_1(\tilde{\theta})$ for all farmers $\tilde{\theta} \neq \theta$ and leave those farmers' constraints unchanged. In sum, we have constructed an alternative contract that satisfies all constraints and achieves a strictly higher objective than the original optimal contract, a contradiction. We conclude that an optimal contract satisfies $\boldsymbol{\pi}_1(\theta) = \boldsymbol{\pi}^*(\boldsymbol{Y}(\theta))$.

We now show that an optimal contract satisfies $\boldsymbol{\pi}_2 = \boldsymbol{\pi}^*(\boldsymbol{\Pi}_1(\boldsymbol{\Pi}))$. Suppose that $\boldsymbol{\pi}_2 \neq \boldsymbol{\Pi}_1(\boldsymbol{\Pi})$. Since $\boldsymbol{\pi}^*(\boldsymbol{\Pi}_1(\boldsymbol{\Pi}))$ maximizes scale for the endowment set $\boldsymbol{\Pi}_1(\boldsymbol{\Pi})$, we know that $\Delta q = q(\boldsymbol{\pi}^*(\boldsymbol{\Pi}_1(\boldsymbol{\Pi}))) - q(\boldsymbol{\pi}_2) > 0$. Consider an alternative contract that makes artisan 2 pay $\boldsymbol{\pi}^*(\boldsymbol{\Pi}_1(\theta))$ and leaves all other payments unchanged.

From (14), we can increase x_2 by $\Delta q(1+\lambda)^{-1}$ and leave the utility of artisan 3 unchanged. At the same time, we strictly increase the expected utility of the artisan in location 2 and in particular its component $(1+\lambda)x_2 - q(\boldsymbol{\pi}_2)$. From (13) and (17), the alternative contract satisfies the individual rationality constraint of the artisan in location 2. It also strictly increases the objective function.

The alternative contract is feasible since the payment of artisan 2 is feasible by construction and changes to the contract assigned to morning meetings do not affect any feasibility constraints for night meetings. We again have a contradiction and conclude that an optimal contract satisfies $\boldsymbol{\pi}_2 = \boldsymbol{\pi}^*(\boldsymbol{\Pi}_1(\boldsymbol{\Pi}))$.

(Sufficiency) We now show that every collection of payments that maximizes the scale of payment within each individual meeting is part of a contract $(X, \mathbf{\Pi})$ that is optimal for some promised utilities v_3 and $v_1(\theta)$.

Consider a candidate contract that maximizes the scale of payments in individual meetings. We first show that we can find a production X and promised utilities such that the candidate contract satisfies all constraints of the optimization problem. We then show that there is no other contract that satisfies all constraints and yields higher payoff.

The payments from Part 1 satisfy the feasibility constraints (12). Fix v_3 and $v_1(\theta)$ and use (14) to choose production of customized goods X:

$$\begin{aligned} x_1(\theta) &= (1+\lambda)^{-1}\left(v_1(\theta) + q\left(\boldsymbol{\pi}_1(\theta)\right) - E(\boldsymbol{p}'\boldsymbol{y}|\theta)\right), \\ x_2 &= q\left(\boldsymbol{\pi}_2\right) - v_3. \end{aligned}$$

Expected utility for an artisan in location 2 from the candidate contract is therefore

$$\begin{aligned} &E\left(U_2(X, \mathbf{\Pi}; \theta)\right) = \\ &\qquad \lambda E q\left(\boldsymbol{\pi}_2\right) - (1+\lambda)\, v_3 + \frac{\lambda}{1+\lambda} E\left[q\left(\boldsymbol{\pi}_1(\theta)\right)\right] - (1+\lambda)^{-1} E(v_1(\theta) - \boldsymbol{p}'\boldsymbol{y}). \end{aligned}$$

From (13), a contract $(X, \mathbf{\Pi})$ satisfies the individual rationality constraint (17) if and only if $E\left(U_2(X, \mathbf{\Pi}; \theta)\right) \geq 0$ and $(1+\lambda)\, x_2 - q\left(\boldsymbol{\pi}_2\right) \geq 0$. Substituting, we obtain

$$\begin{gathered} \frac{\lambda}{1+\lambda} q\left(\boldsymbol{\pi}_2\right) \geq v_3 \\ \frac{\lambda}{1+\lambda} q\left(\boldsymbol{\pi}_2\right) + \frac{\lambda}{(1+\lambda)^2} E\left(q\left(\boldsymbol{\pi}_1(\theta)\right)\right) \geq v_3 + \frac{1}{(1+\lambda)^2} E(v_1(\theta) - \boldsymbol{p}'\boldsymbol{y}). \end{gathered} \tag{27}$$

The conditions are satisfied for example for $v_3 = 0$ and $v_1(\theta) = E(\boldsymbol{p}'\boldsymbol{y}|\theta)$. Moreover, the candidate contract has $\boldsymbol{\pi}_2 = 0$ only in the degenerate case where all endowments are zero for sure, in which case $q\left(\boldsymbol{\pi}_1(\theta)\right) = x_1(\theta) = 0$. The candidate contract thus satisfies the individual rationality constraint (18).

We have shown that the contract satisfies all constraints for some $v_1(\theta)$ and v_3. To show that it is optimal, suppose that there is some alternative feasible and individually rational contract $(\tilde{X}, \tilde{\mathbf{\Pi}})$ that yields strictly higher expected utility for the artisan in location 2 while keeping all other agents' utility levels the same. Suppose that alternative contract assigns different bilateral contracts to meetings between artisans from location 2 and farmers of types $\theta \in \tilde{\Theta} \subset \Theta$.

The payment $\boldsymbol{\pi}^*\left(\boldsymbol{Y}\left(\theta\right)\right)$ is part of any constrained Pareto optimal contract in any such meetings. Since farmer utility is held fixed, it must be that expected utility of an artisan 2 from trade with farmers $E\left(q\left(\tilde{\boldsymbol{\pi}}_1\left(\theta\right)\right)-\tilde{x}_1\left(\theta\right)\right)$ is less or equal than under the candidate contract.

It follows that expected utility from trade with artisan 3, $\left(1+\lambda\right)x_2-q\left(\boldsymbol{\pi}_2\right)$ is strictly higher under the alternative contract. However from (14) this is impossible, since we have assumed that artisan 3 utility is the same for both contracts. We conclude that the candidate contract is indeed optimal. □

Proof of Proposition 4: The previous proposition states that any optimal contract maximizes the scale of payment in each individual meeting. The proof of that implication (labeled "sufficiency" in the proof above) shows that any contract that maximizes the scale of payment in each individual meeting and also delivers utility promises v_3 and $v_1\left(\theta\right)$ must prescribe production (21)–(22) and must satisfy conditions (19)–(20). □

Proof of Proposition 5: We proceed by backward induction. Consider first night meetings between farmers of type θ and artisans from location 2 for a given contract $\left(\tilde{x}_2,\tilde{\boldsymbol{\pi}}_2\right)$ negotiated in the morning meeting. We restrict attention to histories such that $\tilde{\boldsymbol{\pi}}_2=0$ implies $\tilde{x}_2=0$. Any other history would have the artisan in location 3 work for free, which cannot happen in equilibrium.

Consider utilities if the outcome of night bargaining is $\left(\tilde{x}_1,\tilde{\boldsymbol{\pi}}_1\right)$. If the chain of payments is infeasible (that is, $\boldsymbol{p}'\tilde{\boldsymbol{\pi}}_1\geq\boldsymbol{p}'\tilde{\boldsymbol{\pi}}_2$), then the artisan receives zero and the farmer receives $E\left(\boldsymbol{p}'\boldsymbol{y}|\theta\right)$. Otherwise, the utilities of the farmer and artisan are $\left(1+\lambda\right)\tilde{x}_1-q\left(\tilde{\boldsymbol{\pi}}_1\right)+E\left(\boldsymbol{p}'\boldsymbol{y}|\theta\right)$ and

$$\left(1+\lambda\right)\tilde{x}_2+q\left(\tilde{\boldsymbol{\pi}}_1\right)-q\left(\tilde{\boldsymbol{\pi}}_2\right)-\tilde{x}_1,$$

respectively. The outside option of the farmer is $E\left(\boldsymbol{p}'\boldsymbol{y}\right)$ whereas the outside option of the artisan is zero. To maximize joint surplus, the farmer and artisan want to maximize scale $q\left(\tilde{\boldsymbol{\pi}}_1\right)$ subject to feasibility, that is, they want to choose $\tilde{\boldsymbol{\pi}}_1=\boldsymbol{\pi}_1\left(\theta\right)$, the payment prescribed by any optimal contract.

We now choose the weights $\mu_1\left(\theta\right)$ so that farmer θ's share of surplus at the optimal payments $\boldsymbol{\pi}_2$ and $\boldsymbol{\pi}_1\left(\theta\right)$ is exactly $v_1\left(\theta\right)$:

$$\mu_1\left(\theta\right)=\frac{v_1\left(\theta\right)-E\left(\boldsymbol{p}'\boldsymbol{y}|\theta\right)}{\lambda q\left(\boldsymbol{\pi}_2\right)+\frac{\lambda}{1+\lambda}q\left(\boldsymbol{\pi}_1\left(\theta\right)\right)+v_1\left(\theta\right)-E\left(\boldsymbol{p}'\boldsymbol{y}|\theta\right)}.$$

It follows that artisan's equilibrium production given morning contract $(\tilde{x}_2, \tilde{\boldsymbol{\pi}}_2)$ is

$$x_1^*(\tilde{x}_2, \tilde{\boldsymbol{\pi}}_2, \theta) = \frac{1}{1+\lambda}\mu_1(\theta)\left((1+\lambda)\tilde{x}_2 - q(\tilde{\boldsymbol{\pi}}_2)\right) + (1 + \mu_1(\theta)\lambda)\, q(\boldsymbol{\pi}_1(\theta)).$$

Consider, next, bargaining in morning meetings. If the outcome of morning bargaining is $(\tilde{x}_2, \tilde{\boldsymbol{\pi}}_2)$, the utilities of the artisans from locations 3 and 2 are $q(\tilde{\boldsymbol{\pi}}_2) - \tilde{x}_2$ and

$$\begin{aligned}&(1+\lambda)\tilde{x}_2 + E(q(\boldsymbol{\pi}_1(\theta))) - q(\tilde{\boldsymbol{\pi}}_2) - E\left(x_1^*(\tilde{x}_2, \tilde{\boldsymbol{\pi}}_2, \theta)\right)\\ &\quad= \left(1 - \frac{E(\mu_1(\theta))}{1+\lambda}\right)\left((1+\lambda)\tilde{x}_2 - q(\tilde{\boldsymbol{\pi}}_2)\right) + \frac{\lambda}{1+\lambda}E\left((1-\mu_1(\theta))q(\boldsymbol{\pi}_1(\theta))\right),\end{aligned}$$

respectively. The outside option of artisan 3 is zero, and the outside option of artisan 2 is

$$\frac{\lambda}{1+\lambda}E\left((1-\mu_1(\theta))q(\boldsymbol{\pi}_1(\theta))\right).$$

Since an optimal contract maximizes scale and allows promises $v_1(\theta)$, joint surplus in a morning meeting with $\tilde{\boldsymbol{\pi}}_2 = \boldsymbol{\pi}_2$ is positive.

If we further set $\bar{m}_2 = \min_{\boldsymbol{p}\in\boldsymbol{P}} \boldsymbol{p}'\boldsymbol{\pi}_2$, then there is no other bilateral contract $(\tilde{x}_2, \tilde{\boldsymbol{\pi}}_2)$ that delivers more surplus. Indeed, promising a larger payment $\tilde{\boldsymbol{\pi}}_2$ is precluded by the bound. Promising lower payment reduces beneficial production of customized goods.

It follows that the optimal scale of payment in morning meetings is $\boldsymbol{\pi}_2$. We can select the weight μ_3 so the share of surplus going to the artisan in location 3 at the optimal payment $\boldsymbol{\pi}_2$ is v_3:

$$\mu_3 = \frac{v_3}{v_3 + \left(1 - \frac{E(\mu_1(\theta))}{1+\lambda}\right)(\lambda q(\boldsymbol{\pi}_2) - v_3) + \frac{\lambda}{1+\lambda}E\left((1-\mu_1(\theta))q(\boldsymbol{\pi}_1(\theta))\right)}.$$

The production of customized goods follows as $x_2 = q(\boldsymbol{\pi}_2) - v_3$. □

Proof of Proposition 6: Following the same reasoning as in the proof of Proposition 1, in matches between farmers and artisans at location 2 it is optimal to choose the unit of account such that the entire income of the farmer can be passed on to 2. Thus, if artisan 1 meets farmer $i \in \{A, B\}$, artisan 1 will receive a payment consisting of g units of government IOUs and $1 - g$ units of farm good i. For the reasons articulated in Section 4, it will also be optimal to use a common unit of account in all artisan-artisan matches. As in the proof of Proposition 2, this dominant unit of account should be chosen to maximize the payment that can be passed on from location-2 artisans to other artisans. Proposition 2 is not directly applicable here, because there are now three goods that can serve as unit of account. We can state the problem of choosing the optimal unit of account as:

$$\{u^{IOU},\ u^{A},\ u^{B}\} = \underset{u^A,\, u^B,\, u^{IOU}}{\operatorname{argmax}} \left\{ \min_{i,\boldsymbol{p},p^{IOU}} \left\{ \frac{gp^{IOU} + (1-g)p^{i}}{u^{IOU}p^{IOU} + u^{A}p^{A} + u^{B}p^{B}} \right\} \right\}$$

subject to $u^{IOU} + u^{A} + u^{B} = 1$ and $u^{IOU}, u^{A}, u^{B} \geq 0$. What is being maximized is the minimum of the value of the income of an artisan at location 2 (numerator) relative to the value of the unit of account (denominator), where the index $i \in \{A, B\}$ denotes the identity of the farmer that the artisan meets. Given the symmetric price distribution for goods A and B, it is optimal to set:

$$u^{A} = u^{B} = \frac{1 - u^{IOU}}{2}.$$

The worst-case price realization for $\boldsymbol{p}$ is that the relative price of good i is at the minimum. We therefore have:

$$\begin{aligned}
&\min_{i,\boldsymbol{p},p^{IOU}} \left\{ \frac{gp^{IOU} + (1-g)p^{i}}{u^{IOU}p^{IOU} + u^{A}p^{A} + u^{B}p^{B}} \right\} \\
&\quad = \min_{p^{IOU}} \left\{ \frac{gp^{IOU}/\max\{p^{A}, p^{B}\} + (1-g)\underline{p}}{u^{IOU}p^{IOU}/\max\{p^{A}, p^{B}\} + (1-u^{IOU})\frac{\underline{p}+1}{2}} \right\} \\
&\quad = \begin{cases} \frac{g\overline{p}^{IOU} + (1-g)\underline{p}}{u^{IOU}\overline{p}^{IOU} + (1-u^{IOU})\frac{\underline{p}+1}{2}} & \text{if} \quad u^{IOU} > \tilde{u}^{IOU} \\ g + (1-g)\frac{2\underline{p}}{\underline{p}+1} & \text{if} \quad u^{IOU} = \tilde{u}^{IOU} \\ \frac{g\underline{p}^{IOU} + (1-g)\underline{p}}{u^{IOU}\underline{p}^{IOU} + (1-u^{IOU})\frac{\underline{p}+1}{2}} & \text{if} \quad u^{IOU} < \tilde{u}^{IOU}, \end{cases}
\end{aligned}$$

where $\tilde{u}^{IOU}$ is defined in (24), and in the second line we divide both numerator and denominator by the price of the more expensive farm good.

Now notice that for p^{IOU} fixed, the expression on the right-hand side is monotonic in u^{IOU}. Hence, if it is optimal to set $u^{IOU} > \tilde{u}^{IOU}$, the best choice is $u^{IOU} = 1$, and similarly if $u^{IOU} < \tilde{u}^{IOU}$ is optimal the best choice is $u^{IOU} = 0$. Setting $u^{IOU} = 0$ would be better than $u^{IOU} = \tilde{u}^{IOU}$ if the inequality:

$$\frac{g\underline{p}^{IOU} + (1-g)\underline{p}}{\frac{\underline{p}+1}{2}} > g + (1-g)\frac{2\underline{p}}{\underline{p}+1}$$

held, which can be simplified to:

$$\underline{p}^{IOU} \geq \frac{\underline{p}+1}{2},$$

which is ruled out by condition (23). Similarly, $u^{IOU} = 1$ is better than $u^{IOU} = \tilde{u}^{IOU}$ if the inequality:

$$\frac{g\overline{p}^{IOU} + (1-g)\underline{p}}{\overline{p}^{IOU}} \geq g + (1-g)\frac{2\underline{p}}{\underline{p}+1}$$

holds, which can be solved for:

$$\overline{p}^{IOU} \leq \frac{\underline{p}+1}{2},$$

which is the condition stated in the proposition. □

Proof of Proposition 7: Given the symmetric environment, in the optimal allocation either all mismatched agents pay the transport cost, or none of them do. The resulting environments (after potentially paying the transport cost) are as characterized in the discussion of Example 3 if the cost is paid (i.e., matching only within regions, or "separate currencies") or in the discussion of Example 4 if not (matching across regions, or "currency union"). Comparing the optimal allocations, we see that (apart from the transport cost) the only difference in terms of overall welfare is the consumption of the artisanal good by artisans of type 2. This is because given assumed welfare weights farmers and artisans at location 3 at at reservation utility, the production of location-2 artisans is independent of the regime, and location-2 artisans only consume artisanal goods in the optimal allocation. If the transport cost is paid and all agents match within their region (separate currencies), the optimal payment from artisan 2 to 3 in region θ is the one unit of the farm good θ that 2 receives from the farmer. The individual rationality constraint of artisan 3 then implies that we have

$$x_1^{\text{SEPARATE}} = 1.$$

Under the alternative of a currency union, as in Example 4 the optimal payment from 2 to 3 is $\boldsymbol{\pi}_2 = \left(\frac{1}{1+\overline{p}}, \frac{1}{1+\overline{p}}\right)$. The individual rationality constraint of artisan 3 then gives:

$$x_1^{\text{UNION}} = \frac{2}{1+\overline{p}}.$$

That is, in the currency union the scale of production is reduced to ensure that payments can be made even if matching outside the home region. Taking into account that under separate currencies avoiding matching with the other region involves paying the transport cost τ with probability α, welfare is higher when there is matching across regions (and, hence, there is a currency union) if:

$$(1+\lambda)x_1^{\text{UNION}} \geq (1+\lambda)x_1^{\text{SEPARATE}} - \alpha\tau$$

or:

$$\alpha \geq \frac{(1+\lambda)(\overline{p}-1)}{\tau(1+\overline{p})} \equiv \tilde{\alpha},$$

which is the condition given in the proposition. □

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Supplement to
"Money as a Unit of Account"

Matthias Doepke and Martin Schneider

B Coalition-Proof Contracts

When defining optimal contracts in the main text, we have required only individual rationality. In an environment with multiple bilateral meetings, it is also attractive to rule out joint deviations by a pair of agents within a meeting. We now define coalition-proof contracts and point out that the optimal contract characterized by Proposition 3 is coalition proof.

Consider first morning meetings. We require that for all alternative bilateral contracts $(\tilde{x}_2, \tilde{\boldsymbol{\pi}}_2)$ that are feasible—that is, $\tilde{\boldsymbol{\pi}}_2 \in \boldsymbol{\Pi}_2(\boldsymbol{\Pi})$—agents are no better off than under the economywide contract:

$$\begin{aligned} E\left(U_2(X, \boldsymbol{\Pi}; \theta)\right) &\geq (1+\lambda)\, \tilde{x}_2 - q\left(\tilde{\boldsymbol{\pi}}_2\right) + E\left(q\left(\boldsymbol{\pi}_1(\theta)\right) - x(\theta)\right), \\ U_3(X, \boldsymbol{\Pi}) &\geq q\left(\tilde{\boldsymbol{\pi}}_2\right) - \tilde{x}_2. \end{aligned} \tag{28}$$

Proposition 3 shows that any optimal contract maximizes the scale of payment in meetings between artisans 2 and 3. As a result, there cannot be any fruitful joint deviation and the contract satisfies satisfies (28).

Consider now joint deviations by a farmer of type θ and an artisan from location 2 in a night meeting. For all alternative bilateral contracts $(\tilde{x}_1, \tilde{\boldsymbol{\pi}}_1)$ that allow the artisan to make his already promised payment—that is, $\boldsymbol{p}'\tilde{\boldsymbol{\pi}}_1 \geq \boldsymbol{p}'\boldsymbol{\pi}_2$ for all $\boldsymbol{p} \in \boldsymbol{P}$—we require

$$\begin{aligned} E\left(U_2(X, \boldsymbol{\Pi}; \theta)\right) &\geq (1+\lambda)\, x_2 - q\left(\boldsymbol{\pi}_2\right) + E\left(q\left(\tilde{\boldsymbol{\pi}}_1\right)\right) - \tilde{x}_1, \\ U_1(X, \boldsymbol{\Pi}; \theta) &\geq (1+\lambda)\, \tilde{x}_1 - q\left(\tilde{\boldsymbol{\pi}}_1\right) + E(\boldsymbol{p}'\boldsymbol{y}|\theta). \end{aligned}$$

Again an optimal contract does not allow such a deviation since Proposition 3 says the optimal contract maximizes scale within meetings between farmers and artisans from location 2.

C Endowment Income for Artisans

Proposition 3 assumes that artisans in locations 2 and 3 do not receive their own endowment of tradable goods. Instead, they rely for payment on the endowment of the farmer at the head of their chain. Here we consider an extension where artisans can receive endowment income as well. Suppose that at the beginning every agent learns an endowment type $\theta \in \Theta$. Every chain is then characterized by an endowment vector $(\theta_1, \theta_2, \theta_3)$, where θ_i indicates the type of the agent in location i.

It enough to consider an economy with one endowment type in locations 2 and 3. Since types θ_2 and θ_3 are realized before morning meetings where the first actions in the economy are taken, the overall economy divides into many independent subeconomies according to the realization (θ_2, θ_3). Moreover, the effect of an endowment for the artisan in location 3 affects only that agent's outside option, but does not alter any feasibility constraints.

Consider thus a modified environment in which artisans in location 2 receive an endowment with support $\boldsymbol{Y}_2$. The optimal payment satisfies the recursion in Proposition 3 if we define

$$\boldsymbol{\Pi}_1(\boldsymbol{\Pi}) = (\boldsymbol{\pi}_1(\theta) + \boldsymbol{y}_2 : \theta \in \Theta, \boldsymbol{y}_2 \in \boldsymbol{Y}_2).$$

Indeed, an optimal payment in morning meetings maximizes scale given the set $\boldsymbol{\Pi}_1(\boldsymbol{\Pi})$; adding $\boldsymbol{Y}_2$ only changes the initial set of available bundles. Moreover, an optimal bundle must still maximize scale in night meetings also: the key property exploited in the proof of Proposition 3 is that $\boldsymbol{\pi}_1(\theta)$ maximizes the value of production for any price vector. Higher scale in night meetings thus allows higher scale in morning meetings with or without an extra endowment.

While the recursion remains unchanged with an artisan endowment, the evolution of the optimal unit of account is different. In particular, the optimal unit of account will be tailored more closely to the endowment of artisan 2. Like farmer endowments, artisan endowments are thus a force that can makes the unit of account different across transactions. Of course, if the artisan endowment risk is similar in nature to the risk from meeting farmers, say, because it arises from a second meeting with another farmer, then it will have no effect on the unit of account and only increase the scale.

D Setup with Small Default Costs

In the main text, our results were derived for an economy in which default costs are large, so that only non-contingent contracts can be used. In this appendix, we outline how the framework can be extended to allow for small default costs, in the sense that the cost of breaking promises is sufficiently small for breaking promises to be optimal in some cases. Even so, the unit of account still matters, because an appropriately chosen unit of account is necessary to minimize the cost of settling contracts in the economy.

Most of the results for the large-cost case carry over unchanged to the small-cost setting. The key differences arise when we consider what the optimal unit of account should look like. In the small-default-cost setting, the objective is to minimize the probability of default, but not to avoid default entirely. This implies that the probability of meeting different types of agents becomes an important determinant of the optimal unit of account. Parallel to our analysis in Section 6, this feature can be developed into a model of optimal currency areas, even without relying on the ability (as assumed in Section 6) to switch trading partners ex-post at a cost.

We develop the small-default cost setting more fully in the working paper version of this paper, Doepke and Schneider (2013). Here, we describe the basic setup in the context of the partial-equilibrium setup described in Section 3. To allow for a small default cost, we introduce a distinction between a a promised payment and an actual payment. The future payment from customer to supplier is specified in two parts. The first component is a non-contingent promised payment, namely a vector of farm-good quantities $\boldsymbol{\pi} = (\pi^A,\ \pi^B)'$ (as before). By promising $\boldsymbol{\pi}$ to the supplier, the customer commits to delivering goods $\boldsymbol{\pi}$ at date 2. Unlike in the large-cost case, we now allow for the possibility that a customer may not always be able to deliver on a payment promise $\boldsymbol{\pi}$. To deal with this possibility, the second component of the contracted payment consists of a fully contingent payment $h(\boldsymbol{y}, \boldsymbol{p})$, where $h(\boldsymbol{y}, \boldsymbol{p}) \leq \boldsymbol{p}'\boldsymbol{\pi}$, i.e., the value of the alternative payment is no greater than the original promise. The actual payment that the customer has to make in state $(\boldsymbol{y}, \boldsymbol{p})$ is the smaller of the promised and the alternative payment:

$$\min\left\{\boldsymbol{p}'\boldsymbol{\pi}, h(\boldsymbol{y}, \boldsymbol{p})\right\}.$$

Given that $h(\boldsymbol{y}, \boldsymbol{p}) \leq \boldsymbol{p}'\boldsymbol{\pi}$, the actual payment is in fact always equal to $h(\boldsymbol{y}, \boldsymbol{p})$. This actual payment is fully enforced. The full contract between customer and supplier specifies the customized artisinal good x to be produced by the supplier at date 1and delivered to the customer at date 2, the payment promise $\boldsymbol{\pi}$ and actual payment $h(\boldsymbol{y}, \boldsymbol{p})$.

Given that the actual payment is fully contingent, the two-part payment specification as such does not constitute a deviation from complete markets. However, we assume that making a payment that is different from the initial promise is costly. If the promise is met, the customer's cost for settling the contract is zero, $s = 0$. In contrast, whenever we have $h(\boldsymbol{y}, \boldsymbol{p}) < \boldsymbol{p}'\boldsymbol{\pi}$, the customer faces a fixed cost $s = \kappa \geq 0$ in terms of time at date 2. The interpretation is that enforcing the contract and executing the alternative payment in the case of a broken promise involves a legal cost. For different values of κ, this setup captures the usual complete-market setting ($\kappa = 0$), fully non-contingent contracts ($\kappa = \infty$) as in the main analysis above, and settings where the contracting friction affects outcomes, but is not sufficiently strong to reduce to the non-contingent case.