Money as a Unit of Account

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Question

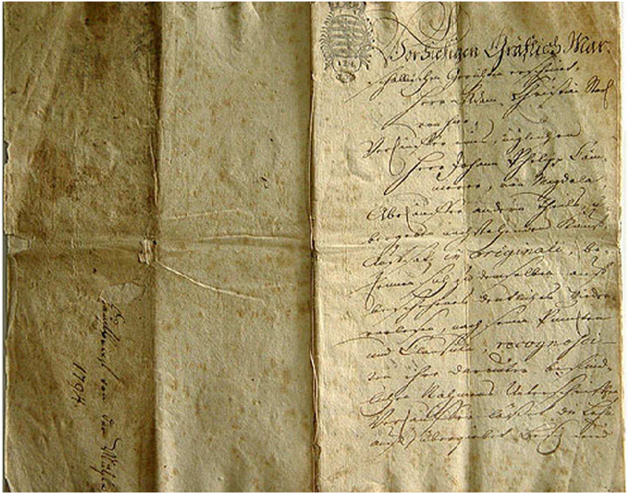
- Explain emergence of a common unit of account for future payments.
  - Why coordinate on a common unit of account?
  - What should be the unit of account?

Examples

Treasury Debt, 2002: U.S. Dollars

Examples

Mühlenerbzins, 1794: Meissnische Gulden, bushels of bran



History

- Unit of account often different from medium of exchange.
- Accounting currencies:
  - Distinct from any existing medium of exchange.
  - Livre tournois in France, ECU in Europe.
- Common unit of account in areas with intensive trade:
  - Many currencies used for payment, contracts mostly in one.
  - Vereinsthaler in Northern Germany before unification.
  - Use of dollar denominated contracts in world trade.
- Government-issued fiat money as unit of account:
  - More common recently as governments borrow more ...
  - ... but not when value too uncertain (dollarization).

Why Coordinate?

- Candidates for unit of account:
  - Goods or assets with quoted prices.
- Three features lead to dominant unit of account:
  1. Cost of breaking promises.
     - Demand for unit of account that hedges relative-price risk.
  2. Trade along credit chains.
     - Demand for common unit of account in chains.
  3. Sequential formation of trading networks.
     - Demand for dominant unit of account in entire economy.

What Should Be the Unit of Account?

- Properties of dominant unit of account:
  - Stable in value relative to revenue of borrowers in many transactions.
  - If government is large, government debt works well ...
  - ... but only if value of debt is stable.
  - In areas with a lot of trade, common unit of account is useful: "currency areas."

Literature

- Hedging through denomination of (bilateral) contracts:
  - Bohn (1988), Neumeyer (1999), Schneider-Tornell (2004), Burnside-Eichenbaum-Rebelo (2006) ...
- Credit chains:
  - Kiyotaki-Moore (2001), ...
- Coordination on indexation:
  - Cooper (1990), Acemoglu (1995).
- Medium of exchange and unit of account:
  - Freeman-Tabellini (1998).
- Matching and currency areas:
  - Matsuyama-Kiyotaki-Matsui (1993), Trejos-Wright (2001), Rey (2003) ...
- Redistribution effects of inflation:
  - Bohn (1990), Doepke-Schneider (2006), Auclert (2006), Doepke et al. (2017) ...

Outline

- General setup.
- Large default cost and divisible projects:
  - Noncontingent contracts, no default, inefficient production.
  - Unit of account maximizes scale of production.
  - Application to government IOUs.
  - Application to optimal currency areas.
- Small default cost and indivisible projects (not today):
  - Contingent contracts, costly default, efficient production.
  - Unit of account minimizes default costs.

Model: Agents, Dates, Goods

- Continuum of agents: Farmers and artisans.
  - Meet and write contracts at date 0.
  - Work at date 1.
  - Exchange goods and consume at date 2.
- Goods:
  - Farm goods: Traded in spot markets at date 2.
  - Artisanal goods: Tailored to matched customer.
  - Labor.

Model: Preferences

- Utility function:

$$u_i(c, x, h) = u(\mathbf{c}) + (1 + \lambda)x - h.$$

$u(\mathbf{c})$: Homogeneous utility derived from vector of farm goods $\mathbf{c}$.
x: Customized artisanal good.
$h \leq 1$: labor supply.

Model: Farm Endowments and Farm-Goods Market

- Farmer type $i \in \{A, B\}$ with mass 0.5 each.
- Farmer of type i endowed with one unit of farm good i at date 2.
- Farm good i trades in spot market at date 2 at price p_i.
- Price risk: Price of farm good i is random.
- Vector of farm-good prices $\mathbf{p} \in \mathbf{P}$ is only source of aggregate risk.
- Prices and units of measurement normalized such that utility is linear in wealth and $E(p_i) = 1$.

Model: Artisan Technology

- Mass one each of artisans at location $i \in [1, 2, \ldots, N]$ along highway.
- One unit of labor at date 1 makes one unit of customized artisanal good at date 2.
- Artisans of type 1 produce for farmers, artisans of type $i + 1$ produce for artisans of type i.
- Artisanal good valuable only for matched customer.
- Artisanal goods do not trade in spot market and do not have a quoted market price.

Model: Matching Process

- Farmers and artisans linked in chains along the highway:

$$\text{Farmer} \longleftarrow 1 \longleftarrow 2 \longleftarrow 3 \longleftarrow 4 \longleftarrow \ldots \longleftarrow N\,.$$

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- Chains created at date 0 by random matching:
  - Morning: Odd *i* artisans travel east and contract with supplier.
  - Night: Odd *i* artisans travel west and contract with customer.
  - Matching risk: Identity of farmer in chain unknown in morning.

Model: Contracts

- In every meeting, customer and supplier can enter into contract specifying:
  1. Quantity $x = h$ to be produced by supplier in period 1 and delivered in period 2.
  2. Payment from customer to supplier in spot market in period 2.
- Introduce friction that favors simple (non-contingent) payment promise:
  - Contract consists of both non-contingent promise and (possibly lower) contingent actual payment.
  - Settling cost if actual payment is lower than promise.
  - Today: Settling cost is infinite: non-contingent promise.

Model: Contracts

- Promise of payment $\boldsymbol{\pi}_{i,j}$:
  - Fixed, non-contingent vector of farm goods.
- Unit of account: Denomination of the promise.

$$\boldsymbol{\pi}_{i,j} = q_{i,j} \begin{pmatrix} u_{i,j} \\ 1 - u_{i,j} \end{pmatrix}.$$

Planning Approach

- To define equilibrium would need to pin down:
  - Bargaining process.
  - Expectations over contracts in other matches.
- Instead, adopt planning approach:
  - Find system of contracts that maximizes total welfare.
  - Planner chooses (among other things) unit(s) of account for promises.
  - Social optimum is an equilibrium for a specific distribution of bargaining power.

Planning Problem

- Maximizing equally weighted welfare is equivalent to maximizing production of artisanal goods.
- Maximization subject to payment feasibility of payments:
  - If i is artisan with costumer g and supplier j, for any $\mathbf{p}$:

$$\mathbf{p}'\boldsymbol{\pi}_{g,i} \geq \mathbf{p}'\boldsymbol{\pi}_{i,j}.$$

  - If i is farmer with supplier j, for any $\mathbf{p}$:

$$p_i \geq \mathbf{p}'\boldsymbol{\pi}_{i,j}.$$

- Maximization also subject to participation constraints.

Examples for Setup with Large Default Cost

- Assumption on farm good prices:
  - Symmetric price distribution.
  - Lower bound of relative price $\underline{p} = \min \{p_i/p_{-i}\} < 1$ independent of i.
  - Upper bound of relative price $\overline{p} = 1/\underline{p} > 1$ independent of i.

One Farmer, One Artisan: Customized Unit of Account

- One type of farmer and one type of artisan:

$$A \longleftarrow 1.$$

- One stage of matching. Price risk only.
- Decide on $x_A = h_1$ and $\boldsymbol{\pi}_{A,1} = q_{A,1}(u_A, 1 - u_A)'$.
- Constraints:
  - Payment feasibility: for all $\mathbf{p} \in \mathbf{P}$,

$$p_a \geq \mathbf{p}'\boldsymbol{\pi}_{A,1}$$

  - Participation constraints:

$$\begin{aligned} 1 - q_{A,1} + (1+\lambda)x_A &\geq 1, \\ q_{A,1} - x_A &\geq 0. \end{aligned}$$

One Farmer, One Artisan: Customized Unit of Account

- Can achieve first-best production:
  - Set artisanal production to $x_A = 1$.
  - Make promise in terms of the farmer's good: $u_{A,1} = 1$.
  - Scale q_A of payment then has to satisfy:

$$\begin{aligned} p_A &\geq p_A q_{A,1}, \\ 1 - q_{A,1} + 1 + \lambda &\geq 1, \\ q_{A,1} - 1 &\geq 0. \end{aligned}$$

  - Hence, $q_{A,1} = 1$.
- Could not get first-best production with other unit of account.

One Farmer, Two Artisans: Unit of Account Passed On

- One type of farmer and two types of artisans:

$$A \longleftarrow 1 \longleftarrow 2.$$

- Two stages of matching. Price risk only.
- Can still achieve first best:
  - Set $x_A = x_1 = 1$.
  - Set $u_{A,1} = u_{1,2} = 1$.
  - Scales of payments need to satisfy:

$$q_{A,1} = q_{1,2} = 1.$$

Two Farmers, Two Artisans: Dominant Unit of Account

- Highway with two types of farmer and two types of artisan:

$$\begin{pmatrix} A \\ B \end{pmatrix} \longleftarrow 1 \longleftarrow 2.$$

- Two stages of matching. Both price and matching risk.
- Problem: In morning matches of 1 and 2, always possible that night partner of 1 (A or B) will not correspond to the chosen unit of account.
- Scale of production needs to be lowered to avoid default.

Two Farmers, Two Artisans: Dominant Unit of Account

- Consider optimal choice of unit of account u, where:

$$\boldsymbol{\pi}_{1,2} = q_{1,2} \begin{pmatrix} u \\ 1-u \end{pmatrix}.$$

- The optimal u solves:

$$u = \underset{u}{\operatorname{argmax}} \left\{ \min_{\mathbf{p}} \left\{ \frac{p_i}{p_A u + p_B(1-u)} \right\} \right\}.$$

- Under symmetric price distribution have:

$$\min_{\mathbf{p}} \left\{ \frac{p_i}{p_A u + p_B(1-u)} \right\} = \frac{\underline{p}}{\max\{u, 1-u\} + \underline{p} \min\{u, 1-u\}},$$

- Thus, optimal unit is $u = 0.5$: Equally weighted bundle of farm goods.

Two Farmers, Four Artisans: Dominant Unit of Account

- Highway with two types of farmer and four types of artisan:

$$\begin{pmatrix} A \\ B \end{pmatrix} \longleftarrow 1 \longleftarrow 2 \longleftarrow 3 \longleftarrow 4.$$

- Optimal to use equally weighted bundle ($u = 0.5$) in 3-4 morning matches as well.
- Without coordination on dominant unit of account, additional sources of mismatch, resulting in lower scale of production.

Extensions

- Income risk for farmers: place more weight on good with higher income risk.
- Price distribution not symmetric: farm goods with less volatile prices are better unit of account.
- Small default costs: use unit of account to minimize probability of default.

Decentralization

- Optimal allocation can be decentralized with Nash bargaining at each stage.
- Unit of account is independent of bargaining weights.
- Bargaining weights matter for distribution of surplus across farmers and artisans.

Government Debt and the Optimal Unit of Account

- Model shows that dominant unit of account is optimal.
- In reality, why is money often used, as opposed to a commodity bundle?
- Introduce government that issues IOUs.
- Will private contracts be denominated in government IOUs?

Government Debt and the Optimal Unit of Account

- In period 0, government buys fraction g of farmers' output in exchange for g units of government IOUs.
- IOU is claim on tax revenue T. Tax revenue is realized at end of date 2, after spot market closes, but before consumption takes place.
- At start of period 2, news about T arrives. IOUs trade in spot market at price:

$$p_{IOU} = E_2(T).$$

Government Debt and the Optimal Unit of Account

- Assume symmetric distribution for p_A and p_B.
- p_{IOU} symmetric with respect to p_A and p_B.
- At extremes of the relative price distribution,

$$\frac{p_{IOU}}{\max\{p_A, p_B\}} \in [\underline{p}_{IOU}, \overline{p}_{IOU}],$$
$$\underline{p}_{IOU} < \frac{\underline{p}+1}{2}.$$

- IOUs can serve as unit of account:

$$\boldsymbol{\pi}_{i,j} = \begin{pmatrix} \pi_{i,j}^{IOU} \\ \pi_{i,j}^{A} \\ \pi_{i,j}^{B} \end{pmatrix} = q_{i,j} \begin{pmatrix} u_{i,j}^{IOU} \\ u_{i,j}^{A} \\ u_{i,j}^{B} \end{pmatrix}$$

Government Debt and the Optimal Unit of Account

- Optimal unit of account:
  - If $\overline{p}_{IOU} < \frac{\underline{p}+1}{2}$, choose IOUs: $u^{IOU} = 1$.
  - Else, choose:

$$u^{IOU} = \frac{g}{g + (1-g)\frac{2\underline{p}}{\underline{p}+1}},$$

$$u^{A}_{i,j} = u^{B}_{i,j} = \frac{1 - u^{IOU}_{i,j}}{2}.$$

- Interpretation: "dollarization" when inflation becomes too volatile.

Optimal Currency Areas

- Consider model in which there are two locations/countries:

$$\begin{array}{ll} \text{Country A:} & A \longleftarrow 1 \longleftarrow 2 \longleftarrow 3 \longleftarrow 4 \\ \text{Country B:} & B \longleftarrow 1 \longleftarrow 2 \longleftarrow 3 \longleftarrow 4 \end{array}$$

- At each stage of matching, probability $x < 0.5$ of meeting someone from the other country.
- If matched in "wrong" country, can pay cost to rematch.
- Should a common unit of account be adopted?

Optimal Currency Areas

- Separate units (A for A, B for B):
  - Maximizes production conditional on matching within one country.
  - But requires paying rematch cost to avoid possibility of default.
- Common unit of account:
  - Some ex-post risk due to meeting partners from either country.
  - But no need to pay rematch cost.
- Common unit optimal when x sufficiently large.
- Common unit more attractive when chains of credit are longer.

Summary

- Three features lead to common unit of account:
  1. Cost of breaking promises.
  2. Trade along credit chains.
  3. Sequential formation of trading networks.
- Properties of optimal unit of account:
  - Stable in value relative to revenue of borrowers in many transactions.
  - Government debt works well if large and not too volatile.
  - Common "currency areas" optimal if lots of trade.

Next Steps

- Explain history of units of accounts and currency areas.
- Examine role of financial intermediaries.
- Examine costs of monetary instability.

Setup with Small Default Cost

- Discrete labor supply $h \in \{0, 1\}$.
- Small default costs: $\kappa < \lambda$.
- Everyone works under optimal allocation.
- Maximize surplus by minimizing probability of default.
- Do this by coordinating on a dominant unit of account.
- Intuition as in large-default-cost case, but rather than extremes of price distribution, probability of default matters.

Optimal Contract

- All agents work: $h_i = 1$ for all i.
- Farmers promise and pay their entire harvest.
- Choose promise $\boldsymbol{\pi}$ in matches between artisans to maximize:

$$E\left[\Pr\left[p_h\left(1+\lambda\right) \geq \mathbf{p}'\boldsymbol{\pi}\right]\right]$$

subject to:

$$E\left[\min\left\{p_h\left(1+\lambda\right), \mathbf{p}'\boldsymbol{\pi}\right\}\right] \geq 1.$$

- Actual payment by artisan i in chain headed by farmer h:

$$v_{i,j}(\mathcal{N}, \mathbf{p}) = \min\left\{\mathbf{p}'\boldsymbol{\pi}, p_h\left(1+\lambda\right)\right\}.$$